1. A binary communication system is designed to decide between two hypotheses $H_0$ and $H_1$, where $H_0$ corresponds to the transmission of $s_0(t)$ and $H_1$ corresponds to the transmission of $s_1(t)$. The decision between $H_0$ and $H_1$ is to be made based on $N$ independent observations (samples) of the received signal $y(t) = s_i(t) + n(t)$ taken during the signaling interval of duration $T$. Assume that $n(t)$ has a flat PSD of $N_0/2$ and is band-limited to $B$ Hz. Also, assume that $s_0(t)$ and $s_1(t)$ are antipodal rectangular pulses of duration $T$ and amplitude $\sqrt{P}$, that are transmitted with probabilities $Pr(H_0)$ and $Pr(H_1)$, respectively.

(a) What is the maximum value of $N$ such that the samples of the received signal $y(t)$ are independent? Express your answer in terms of $B$ and $T$.

(b) At each sample time, $y_i = y((i + 1/2)T_s)$ is compared to a threshold $\gamma$ and the following per sample correct decision probabilities are defined
$$\rho = Pr(y_i \geq \gamma | H_1) = Pr(y_i < \gamma | H_0)$$

The final decision on which hypothesis is true is made according to the following rule:
- $\rightarrow H_1$ if there are $\lambda$ or more samples, $y_i$, that exceed the threshold $\gamma$.
- $\rightarrow H_0$ if there fewer than $\lambda$ samples, $y_i$, that exceed the threshold $\gamma$.
Write an expression for the two conditional error probabilities $Pr(\rightarrow H_1 | H_0)$ and $Pr(\rightarrow H_0 | H_1)$.

(c) Write an expression for the average probability of error.

(d) Suppose now that, rather than making tentative decision at each sample, the $N$ independent samples are summed and a final decision is made as follows:
- $\rightarrow H_1$ if the sum of all samples exceed the threshold $\Lambda$.
- $\rightarrow H_0$ if the sum of all samples does not exceed the threshold $\Lambda$.
Write an expression of the average probability of error in this case. Express your answer in terms of $P$, $T$, $N_0$, $N$, $\Lambda$, and the a priori probabilities $Pr(H_0)$ and $Pr(H_1)$.

(e) How should $\Lambda$ be chosen so as to make the conditional error probabilities, $Pr(\rightarrow H_1 | H_0)$ and $Pr(\rightarrow H_1 | H_0)$, equal.
2. A BPSK signal is transmitted. The received signal is given by

\[ x_r(t) = A d p(t) \cos(\omega_c t + \theta) \]

where \( Pr(d = 1) = Pr(d = -1) = \frac{1}{2} \), and

\[ p(t) = \begin{cases} 
1 & 0 < t \leq T \\
0 & \text{otherwise} 
\end{cases} \]

Using the receiver shown below:

Assume \( n_0(t) \) and \( n_1(t) \) are independent zero mean Gaussian processes with flat PSDs \( \frac{N_0}{2} \) and \( \frac{N_0}{2} \) respectively.

(a) Determine the structure of the optimal decision device that results in the minimum probability of error.

(b) Calculate the probability of error assuming equiprobable signaling and equal noise powers.
3. Consider the system shown below.

Assuming the input to the system is

\[ x_r(t) = n_w(t), \]

where \( n_w(t) \) is a zero mean Gaussian noise process with a double-sided PSD of \( N_o/2 \). Find the power spectral density of the output \( y_d(t) \).

4. A BPSK signal is transmitted. The received signal is given by

\[ x_r(t) = A d p(t) \cos(\omega_c t + \theta) \]

where \( Pr(d = 1) = Pr(d = -1) = \frac{1}{2} \), and

\[ p(t) = \begin{cases} 1 & 0 < t \leq T \\ 0 & \text{otherwise} \end{cases} \]

Using the receiver shown below:

(a) Calculate the test statistic \( g \).

(b) Using the decision device

\[ \text{if } g \geq 0 \quad \text{decide } \hat{d} = 1 \]
\[ \text{if } g < 0 \quad \text{decide } \hat{d} = -1 \]

calculate the probability of bit error for the system; write your answer in terms of the received energy-per-bit, \( E_b \).