1. A sinusoidal signal with an amplitude $A$ is added to Gaussian narrow-band noise, $n(t)$, where $\text{Var}[n(t)] = \sigma^2$,

$$A \cos(\omega_c t) + n(t).$$

Find the probability density function of its magnitude, measured with respect to the sinusoid (the probability density function of the magnitude of the low-pass equivalent signal).

2. A binary digital communication system transmits

$$x_r(t) = Adp(t) \cos(\omega_c t)$$

where $d$ takes on values one or zero with probability $Pr(d = 0) = Pr(d = 1) = 1/2$. This is known as On-Off Keying since the signal is either present or not. The non-coherent receiver shown below receives

$$x_r(t) = Adp(t) \cos(\omega_c t + \theta).$$

where $\theta$ is modeled by a random variable which is uniformly distributed between $[0, 2\pi)$.

(a) Calculate the test statistics $g_c$ and $g_s$.

(b) Assuming a large input signal-to-noise ratio, design the decision device to minimize the probability of error.

You may use the following approximation to the modified Bessel function,

$$I_0(z) \approx \frac{e^z}{\sqrt{2\pi z}} \quad z >> 1$$

(Hint: $x - \ln(\sqrt{x})$ is approximately $x$ when $x >> 1$)
(c) Using the decision device from (b) calculate the probability of bit error for the system (you may again assume a large signal-to-noise ratio). Write your answer in terms of the received energy-per-bit, $E_b$.

(d) Compare the performance of non-coherent On-Off Keying with the performance of coherent On-Off Keying from homework 1 (plot out the performance for $E_b/N_o$ ranging from 0 to $15 dB$.}
3. An 8-FSK system uses the following 8 orthogonal waveforms

\[
\begin{align*}
    f_0(t) &= p(t) \cos(\omega_0 t) \\
    f_1(t) &= p(t) \cos(\omega_1 t) \\
    &\vdots \\
    f_7(t) &= p(t) \cos(\omega_7 t)
\end{align*}
\]
equiprobably, where the \( \omega_i \) are chosen such that there is no (or minimal) spectral overlap between the signals. The signals are thus orthogonal

\[
\int_{-\infty}^{\infty} f_i(t)f_j(t)dt = E_f \delta_{ij}.
\]

The transmitted signal is

\[
x_t(t) = A_t f_i(t) \cos(\omega_c t)
\]

and the received signal is

\[
x_r(t) = A f_i(t) \cos(\omega_c t + \theta),
\]

where \( \theta \) is uniformly distributed in \([0, 2\pi)\). This is an example of an M-ary system whose probability of symbol error, using coherent detection, was derived in class. A non-coherent 8-FSK receiver is shown below.

Derive the probability of symbol error for this non-coherent 8-FSK receiver in terms of the average received energy-per-bit, \( E_b \); your final answer should be a closed-form expression (hint: use the binomial expansion).
4. M-ary PSK has a transmitted waveform given by

\[ x_t(t) = A_t p(t) \cos \left( \frac{2\pi}{M} \left( m - 1 \right) \right) \cos(\omega_c t) - A_t p(t) \sin \left( \frac{2\pi}{M} \left( m - 1 \right) \right) \sin(\omega_c t) \]

where \( m \) is a discrete R.V. uniformly distributed between \([1, M]\).

(a) Draw out the constellation for 8-PSK.

(b) Draw the block diagram of a coherent matched filter M-PSK receiver. Assume the front end receiver noise is modeled as AWGN with a PSD of \( N_0/2 \).

(c) Find the test statistics prior to the decision device (i.e. \( g_e \) and \( g_s \))

(d) Draw the optimal decision regions for 8-PSK.

(e) Find the probability of symbol error for 8-PSK. The following, high SNR, approximation can be used to obtain a closed form expression

\[
\frac{1}{\sqrt{2\pi}} \int_0^\infty re^{-\frac{(r-m)^2}{2}} dr \approx m \quad m \gg 1.
\]