Synchronization

Consider the more general AWGN channel model

$$r_c(t) = \alpha v_c(t-\tau) + w_c(t)$$

$v_c(t)$ – transmitted bandpass signal
$w_c(t)$ – bandpass AWGN signal
$r_c(t)$ – received bandpass signal
τ – transmission delay
α – channel attenuation

In general, the transmission delay and channel attenuation are unknown, and their effects must be taken into consideration.

Carrier Recovery

Suppose

$$v_c(t) = \mathbf{Re}\left\{v(t)\sqrt{2}e^{j2\pi f_c t}\right\}$$

is transmitted, where $v(t)$ is the complex lowpass equivalent transmitted signal. The received signal is

$$\begin{aligned} r_c(t) &= \alpha v_c(t-\tau) + w_c(t) \\ &= \alpha \mathbf{Re}\left\{v(t-\tau)\sqrt{2}e^{j2\pi f_c (t-\tau)}\right\} + w_c(t) \\ &= \alpha \mathbf{Re}\left\{v(t-\tau)\sqrt{2}e^{j2\pi f_c t}e^{j\phi_c}\right\} + w_c(t) \end{aligned}$$

where $\phi_c = -2\pi f_c \tau$ is the carrier phase uncertainty introduced by the unknown transmission delay.
If the transmission delay could be estimated extremely accurately, then it would be possible to estimate the carrier phase uncertainty. However, since the carrier frequency is usually very large, any slight error in estimating τ will lead to great uncertainty about ϕ_c.

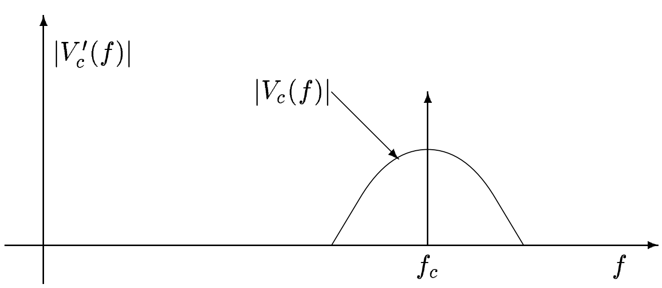
Example: Suppose the carrier frequency is $f_c = 1$ Gz, and the transmission delay is $\tau = 2$ μsec. The carrier phase uncertainty is $\phi_c = -2\pi f_c \tau = -4000\pi = 0$ radians. If the estimate of the transmission delay is $\widehat{\tau} = 2.0005$ μsec, the estimate of the phase uncertainty would be $\widehat{\phi}_c = -2\pi f_c \widehat{\tau} = -4001\pi = \pi$ radians, for an error of π radians.

Furthermore, a carrier phase uncertainty will result if there is a carrier frequency mismatch between the transmitter and receiver. If the carrier phase uncertainty is neglected, reliable data transmission is impossible.
Two basic approaches to carrier recovery are:

1. Pilot signal insertion
   - an unmodulated carrier is transmitted with the data-bearing signal
   - receiver uses narrowband filter to extract the pilot tone
   - use pilot tone for demodulation
   - requires additional power to transmit the pilot.

$$v'_c(t) = v_c(t) + K\cos(2\pi f_c t)$$

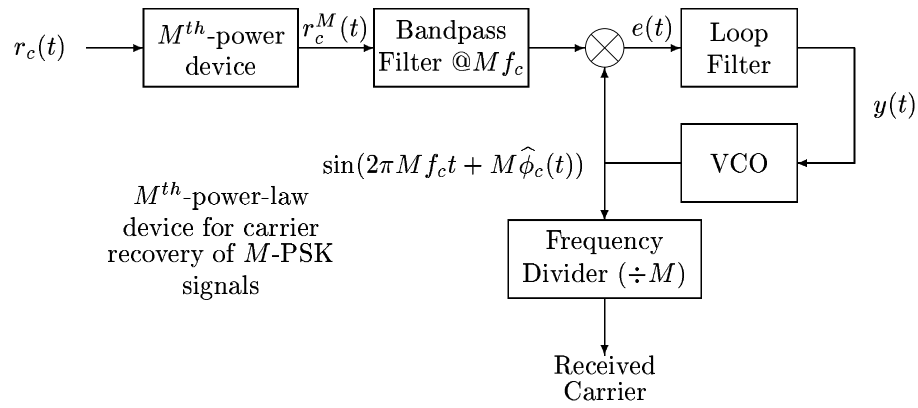
$$V'_c(f) = V_c(f) + \frac{K}{2}\delta(f-f_c) + \frac{K}{2}\delta(f+f_c)$$



2. Suppressed Carrier Extraction
   - extract carrier reference signal from data-bearing signal
   - use phase locked loop (PLL), squaring loop, Costas loop, etc ...

Example:

For M-PSK



M^{th}-power-law device for carrier recovery of M-PSK signals

Ignoring the noise component,

$$\begin{aligned} r_c(t) &= v_c(t) \\ &= \sum_{n=0}^{N_a-1} h_T(t-nT) A \cos(2\pi f_c t + \phi_c + \theta_n) \end{aligned}$$

where $\theta_n = \frac{2\pi}{M} a_n$ is the transmitted phase, ϕ_c is the phase uncertainty, $h_T(t)$ is the transmitted pulse shape, A is an arbitrary signal amplitude, and N_a is the number of symbols transmitted in a packet. The problem is to estimate ϕ_c, which is obscured by the transmitted data.

- M^{th}-power-law device generates harmonics of f_c.

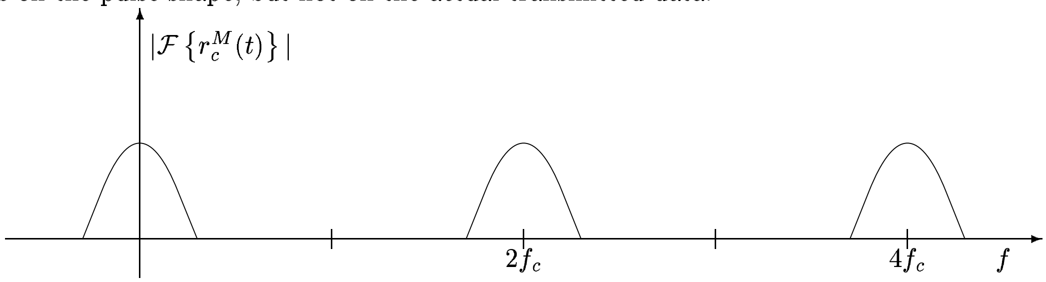
$$\begin{aligned} r_c^2(t) &= \sum_{n=0}^{N_a-1} \sum_{m=0}^{N_a-1} h_T(t-nT) h_T(t-mT) A^2 \cos(2\pi f_c t + \phi_c + \theta_n) \cos(2\pi f_c t + \phi_c + \theta_m) \\ &= \sum_{n=0}^{N_a-1} h_T^2(t-nT) A^2 \cos^2(2\pi f_c t + \phi_c + \theta_n) \\ &= \sum_{n=0}^{N_a-1} h_T^2(t-nT) \frac{A^2}{2} \left[1 + \cos(4\pi f_c t + 2\phi_c + 2\theta_n)\right] \\ r_c^4(t) &= \sum_{n=0}^{N_a-1} \sum_{m=0}^{N_a-1} h_T^2(t-nT) h_T^2(t-mT) \frac{A^4}{4} \left[1 + \cos(4\pi f_c t + 2\phi_c + 2\theta_n)\right] \\ &\qquad \times \left[1 + \cos(4\pi f_c t + 2\phi_c + 2\theta_m)\right] \\ &= \sum_{n=0}^{N_a-1} h_T^4(t-nT) \frac{A^4}{4} \left[1 + \cos(4\pi f_c t + 2\phi_c + 2\theta_n)\right]^2 \\ &= \sum_{n=0}^{N_a-1} h_T^4(t-nT) \frac{A^4}{4} \left[1 + 2\cos(4\pi f_c t + 2\phi_c + 2\theta_n) + \cos^2(4\pi f_c t + 2\phi_c + 2\theta_n)\right] \\ &= \sum_{n=0}^{N_a-1} h_T^4(t-nT) \frac{A^4}{4} \left[1 + 2\cos(4\pi f_c t + 2\phi_c + 2\theta_n) + \tfrac{1}{2} + \tfrac{1}{2}\cos(8\pi f_c t + 4\phi_c + 4\theta_n)\right] \end{aligned}$$

In general,

$$\begin{aligned} r_c^M(t) &= \cdots + \sum_{n=0}^{N_a-1} h_T^M(t-nT) 2 \left(\frac{A}{2}\right)^M \cos(2\pi M f_c t + M\phi_c + M\theta_n) \\ &= \cdots + \sum_{n=0}^{N_a-1} h_T^M(t-nT) 2 \left(\frac{A}{2}\right)^M \cos(2\pi M f_c t + M\phi_c + M\frac{2\pi}{M} a_n) \end{aligned}$$

$$= \cdots + \sum_{n=0}^{N_a-1} h_T^M(t-nT) 2 \left(\frac{A}{2}\right)^M \cos(2\pi M f_c t + M\phi_c)$$
$$= \cdots + x(t)\cos(2\pi M f_c t + M\phi_c)$$

where
$$x(t) = 2\left(\frac{A}{2}\right)^M \sum_{n=0}^{N_a} h_T^M(t-nT)$$
depends on the pulse shape, but not on the actual transmitted data.



- the bandpass filter isolates the harmonic
  $$x(t)\cos(2\pi M f_c t + M\phi_c)$$
- the output of the multiplier is
  $$\begin{aligned} e(t) &= x(t)\cos(2\pi M f_c t + M\phi_c)\sin(2\pi M f_c t + M\widehat{\phi}_c(t)) \\ &= x(t)\tfrac{1}{2}\left[\sin(4\pi M f_c t + M[\phi_c + \widehat{\phi}_c(t)]) + \sin(M[\phi_c - \widehat{\phi}_c(t)])\right] \end{aligned}$$
  where $\widehat{\phi}_c(t)$ is an estimate of the carrier phase uncertainty.
- the loop filter (a narrowband filter) removes the high-frequency component, and most of $x(t)$, leaving
  $$y(t) = K\sin M[\phi_c - \widehat{\phi}_c(t)] \cong KM[\phi_c - \widehat{\phi}_c(t)]$$
- the voltage controlled oscillator (VCO) produces a sinusoid $\sin[\alpha(t)]$, whose instantaneous phase is
  $$\alpha(t) = 2\pi M f_c t + \frac{1}{K}\int_{-\infty}^{t} y(\tau)\, d\tau$$
  where K is a gain constant.
  If at time $t = t_1$,
  $$\alpha(t_1) = 2\pi M f_c t_1 + M\widehat{\phi}_c(t_1)$$
  then at time $t_2 > t_1$
  $$\begin{aligned} \alpha(t_2) &= 2\pi M f_c t_2 + M\widehat{\phi}_c(t_1) + \frac{1}{K}\int_{t_1}^{t_2} y(\tau)\, d\tau \\ &\cong 2\pi M f_c t_2 + M\widehat{\phi}_c(t_1) + \frac{1}{K}\int_{t_1}^{t_2} KM[\phi_c - \widehat{\phi}_c(\tau)]\, d\tau \\ &\cong 2\pi M f_c t_2 + M\widehat{\phi}_c(t_1) + M[\phi_c - \widehat{\phi}_c(t_1)](t_2 - t_1) \end{aligned}$$
  As $t \to \infty$, $\alpha(t) \to 2\pi M f_c t + M\phi_c$.
- the frequency divider output is
  $$\sin\left(2\pi f_c t + \widehat{\phi}_c(t)\right)$$

- as $t \to \infty$, $\widehat{\phi}_c(t) \to \phi_c$, (mod $\frac{2\pi}{M}$)
- the phase tracking loop can lock onto ϕ_c with an offset of any integer multiple of $\frac{2\pi}{M}$.
- these carrier extraction techniques introduce a phase ambiguity that is an integer multiple of $\frac{2\pi}{M}$.

Note: this problem can be overcome by differentially encoding the signal prior to transmission, and differentially decoding the signal at the receiver.

Differentially Encoded M-PSK (M-DPSK)

To overcome the phase ambiguity introduced by the phase tracking loop, differential phase encoding is often used.

- For traditional (absolutely-encoding) M-PSK, if a_n is the symbol transmitted in the n^{th} symbol interval, with $a_n \in \{0, 1, \ldots, M-1\}$, then the phase transmitted in the n^{th} symbol interval is
$$\theta_n = \frac{2\pi}{M} a_n$$
- For diffentially encoded M-PSK, the phase transmitted in the n^{th} symbol interval is
$$\theta_n = \theta_{n-1} + \frac{2\pi}{M} a_n$$
where θ_{n-1} is the phase transmitted in the previous symbol interval.
- Because of the phase ambiguity, the receiver decides that
$$\begin{aligned} \widehat{\theta}_{n-1} &= \theta_{n-1} + \theta_\epsilon \\ \widehat{\theta}_n &= \theta_n + \theta_\epsilon \end{aligned} \qquad \left(\begin{array}{c}\text{ignoring error} \\ \text{due to noise}\end{array}\right)$$
where θ_ϵ is the phase ambiguity, with $\theta_\epsilon \in \left\{ \frac{2\pi}{M} k \,\middle|\, k = 0, 1, \ldots, M-1 \right\}$
- The difference between these two is
$$\begin{aligned} \widehat{\theta}_n - \widehat{\theta}_{n-1} &= \theta_n + \theta_\epsilon - \theta_{n-1} - \theta_\epsilon \\ &= \theta_n - \theta_{n-1} \\ &= \frac{2\pi}{M} a_n \end{aligned}$$
- Taking errors due to noise into account, note that an error in $\widehat{\theta}_n$ will cause not only an error in $\widehat{a}_n$, but also in $\widehat{a}_{n+1}$. As a result, the probability of error for M-DPSK is about twice that of M-PSK.

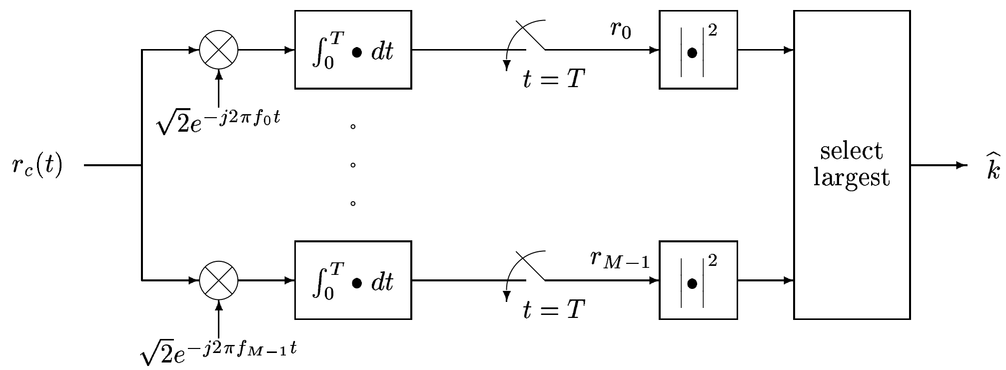
Noncoherent Receivers

An alternative to carrier recovery

- Coherent Receivers - perform carrier recovery
- Noncoherent Receivers - do not perform carrier recovery

Noncoherent receivers can only be used with certain signalling schemes

(a) Noncoherent detection of frequency shift keying



$$\begin{aligned} r_m &= \int_0^T r_c(t)\sqrt{2}e^{-j2\pi f_m t}\, dt \\ &= \int_0^T \left[A\sqrt{2}\cos(2\pi f_k t + \phi_c) + w_c(t)\right] \sqrt{2}e^{-j2\pi f_m t}\, dt \\ &= 2A \int_0^T \cos(2\pi f_k t + \phi_c) e^{-j2\pi f_m t}\, dt + w_m \\ &= A \int_0^T \left[e^{j2\pi f_k t} e^{j\phi_c} + e^{-j2\pi f_k t} e^{-j\phi_c}\right] e^{-j2\pi f_m t}\, dt + w_m \\ &= A \int_0^T e^{j2\pi(f_k - f_m)t} e^{j\phi_c}\, dt + A \int_0^T e^{-j2\pi(f_k + f_m)t} e^{-j\phi_c}\, dt + w_m \\ &= A \frac{e^{j\phi_c}}{j2\pi(f_k - f_m)} \left[e^{j2\pi(f_k - f_m)t}\right]_0^T + A \frac{e^{-j\phi_c}}{-j2\pi(f_k + f_m)} \left[e^{-j2\pi(f_k + f_m)t}\right]_0^T + w_m \end{aligned}$$

$$
\begin{aligned}
&= A\frac{e^{j\phi_c}}{j2\pi(f_k - f_m)}\left[e^{j2\pi(f_k-f_m)T} - 1\right] + A\frac{e^{-j\phi_c}}{-j2\pi(f_k + f_m)}\left[e^{-j2\pi(f_k+f_m)T} - 1\right] + w_m \\
&= A\frac{e^{j\phi_c}}{j2\pi(f_k - f_m)}\left[e^{j\pi(f_k-f_m)T} - e^{-j\pi(f_k-f_m)T}\right] e^{j\pi(f_k-f_m)T} \\
&\qquad + A\frac{e^{-j\phi_c}}{-j2\pi(f_k + f_m)}\left[e^{-j\pi(f_k+f_m)T} - e^{j\pi(f_k+f_m)T}\right] e^{-j\pi(f_k+f_m)T} + w_m \\
&= Ae^{j\phi_c}\frac{\sin(\pi[f_k - f_m]T)}{\pi(f_k - f_m)}e^{j\pi(f_k-f_m)T} + Ae^{-j\phi_c}\frac{\sin(\pi[f_k + f_m]T)}{\pi(f_k + f_m)}e^{-j\pi(f_k+f_m)T} + w_m \\
&= Ae^{j\phi_c}\frac{\sin(\pi[f_k - f_m]T)}{\pi(f_k - f_m)}e^{j\pi(f_k-f_m)T} + w_m
\end{aligned}
$$

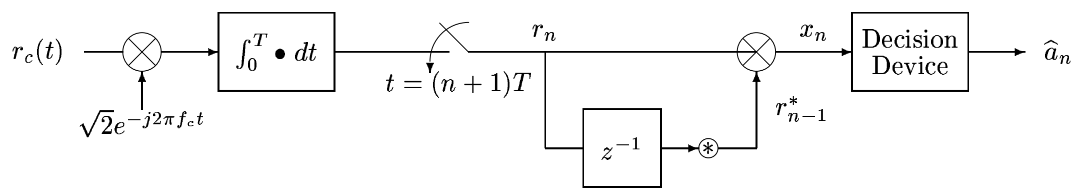
If $f_k - f_m = (k-m)\Delta f_c = \frac{k-m}{T}$ then

$$r_m = Ae^{j\phi_c}\delta_{k-m} + w_m$$

and

$$\left|r_m\right|^2 = A^2\delta_{k-m} + \text{ noise terms}$$

(b) Noncoherent M-DPSK



$$r_n = \sqrt{\mathcal{E}_s}e^{j\theta_n}e^{j\phi_c} + w_n$$

$$
\begin{aligned}
x_n &= r_n r^*_{n-1} \\
&= \mathcal{E}_s e^{j(\theta_n + \phi_c - \theta_{n-1} - \phi_c)} + \text{ noise terms} \\
&= \mathcal{E}_s e^{j\frac{2\pi}{M}a_n} + \text{ noise terms}
\end{aligned}
$$

Timing Recovery (Symbol Synchronization)

The receiver must sample the matched filter outputs at the precise sampling instants, $t_n = nT + \tau$. The receiver must know

- the symbol rate, $1/T$, and
- the transmission delay τ.

For correct sampling, the receiver requires a synchronized clock signal. Several approaches

1. Master Clock
   - transmitter and receiver are synchronized to an external master clock
   - receiver must still estimate and compensate for transmission delay
     - OK if $\tau << T$

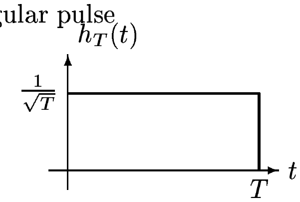
2. Transmitted Clock
   - clock signal is transmitted along with the data
   - receiver uses narrowband filter to extract clock signal from data
   - good timing recovery since clock signal has same delay as data signal
   - requires power to transmit clock, reducing power available for data
   - clock signal requires additional bandwidth

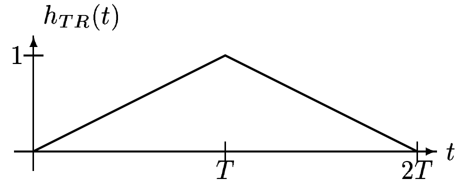
3. Extract clock signal from received data signal

   Early-Late Gate Synchronization

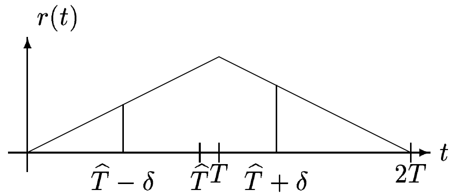
   - one possible approach
   - based on the fact that the matched filter output is at a maximum at the correct sampling instant:

     Example: rectangular pulse

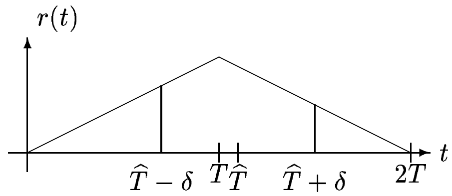




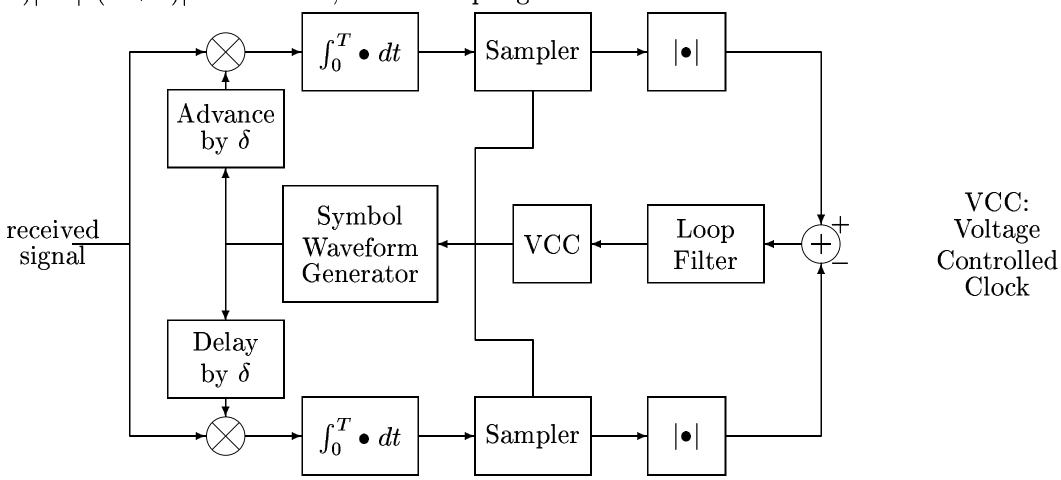
   - if $\widehat{T}$ is an estimate of the correct sampling instant, take two additional samples, one at $\widehat{T} - \delta$ and one at $\widehat{T} + \delta$
   - if $\widehat{T} < T$ then



   - if $\widehat{T} > T$ then



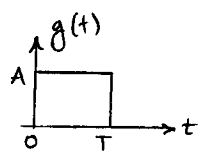
   - If $|r(\widehat{T} - \delta)| < |r(\widehat{T} + \delta)|$ then $\widehat{T} < T$, so the sampling instant should be delayed.
   - If $|r(\widehat{T} - \delta)| > |r(\widehat{T} + \delta)|$ then $\widehat{T} > T$, so the sampling instant should be advanced.

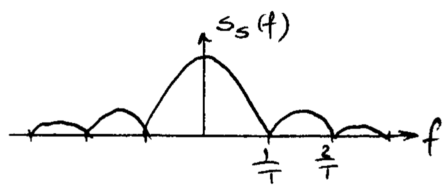


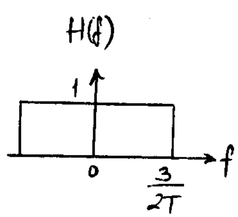
Intersymbol Interference

- occurs when the transmission channel is not ideal.
- In most cases, channel is bandlimited and/or not flat
- In detection design, we consider the ideal channel case. Here, we neglect the effects of the AWGN and focus on the channel distortion.

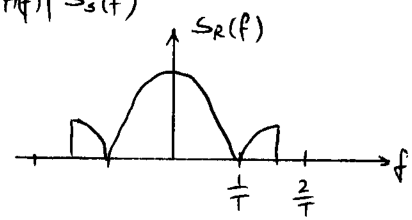
$$s(t) = \Sigma a_k\, g(t-kT) \longrightarrow \boxed{\text{ideal-LPF channel}} \longrightarrow r(t) = \Sigma a_k\, \ell(t-kT)$$

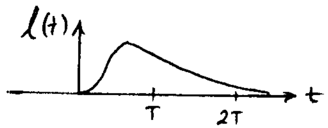


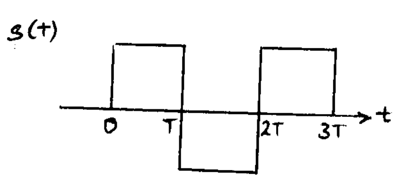




$S_R(f) = |H(f)|^2 S_s(f)$



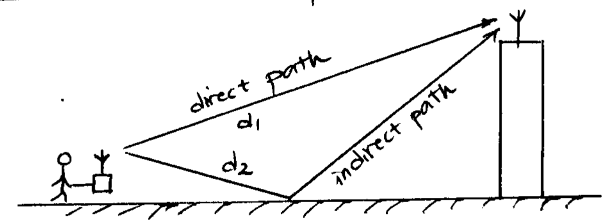




$\Rightarrow$

leakage from one bit to another $\rightarrow$ ISI

Ex: Wireless multipath channel

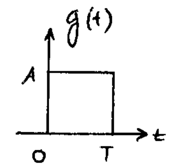


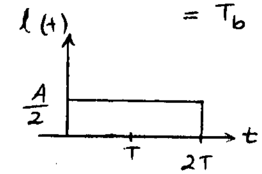
$d_2 - d_1 = 60$ meters

$R_b = 5$ Mbps

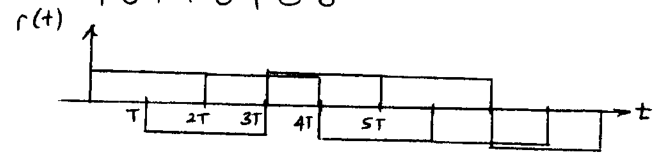
$$h(t) = \frac{1}{2}\delta(t-t_0) + \frac{1}{2}\delta(t-t_0-\tau)$$

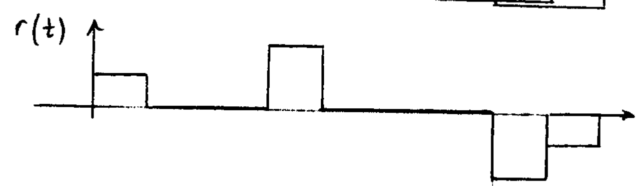
$$\tau = \frac{\Delta d}{c} = \frac{60 \text{ meters}}{3\,10^8 \text{ m/sec}} = 0.2\; 10^{-6} \text{ sec} = T_b$$





1 0 1 1 0 1 0 0



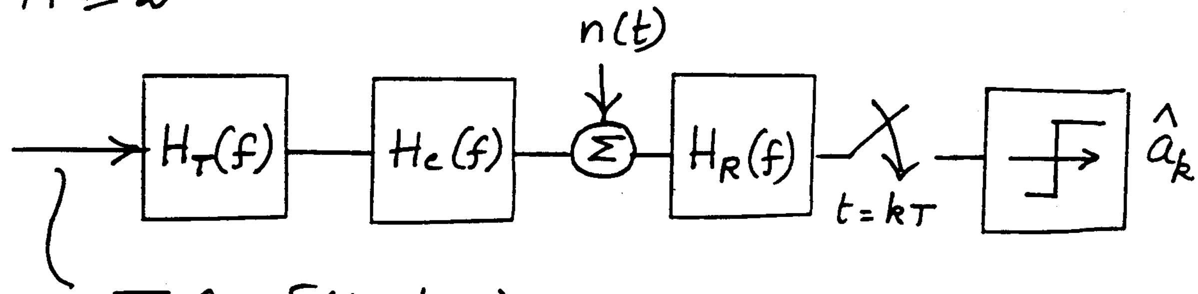


P_e calculation:

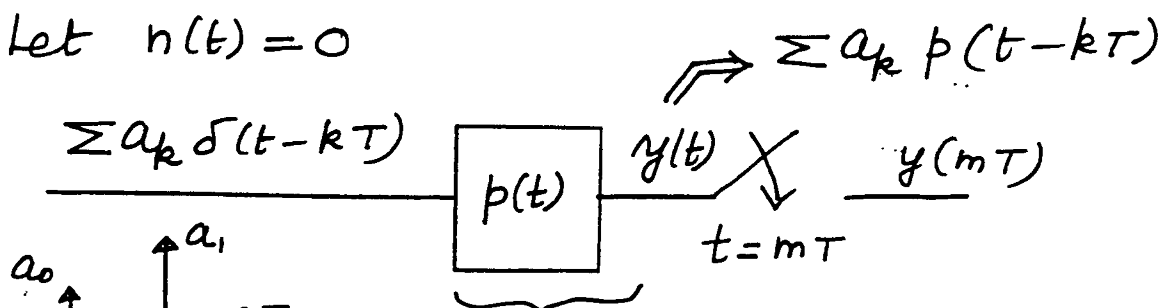
		P(detection error for the 2nd bit)
1	1	$\rightarrow$ 0
1	0	$\rightarrow$ 1/2
0	1	$\rightarrow$ 1/2
0	0	$\rightarrow$ 0

$$P_e = \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot\frac{1}{2} + \frac{1}{4}\cdot\frac{1}{2} + \frac{1}{4}\cdot 0 = \frac{1}{4}$$

NYQUIST PULSE SHAPING

- Transmission thru' bandlimited channels.
- M-ary PAM, symbol rate, $R = 1/T$, $M \geq 2$
- 

- Let $n(t) = 0$

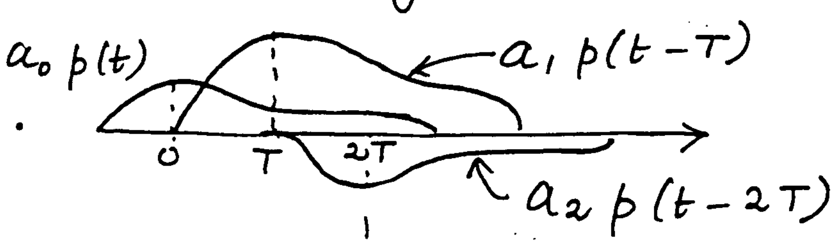


Equivalent channel

$P(f) = H_T(f)\,H_c(f)\,H_R(f)$

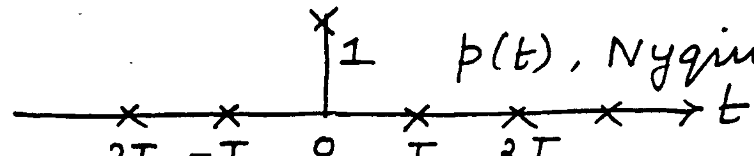
- Perfect digital transmission

$$\Rightarrow \quad y(mT) = a_m$$

- 

$$y(2T) = \underbrace{a_2\,p(0)}_{\substack{\text{desired}\\ \text{value}}} + \underbrace{a_0\,p(2T) + a_1\,p(T)}_{\substack{\text{ISI}\\ \text{(Intersymbol}\\ \text{Interference)}}}$$

- Nyquist's criterion for zero ISI
  $\Rightarrow$ zero-forcing criterion
- $y(mT) = a_m \Rightarrow p(mT) = \begin{cases} 1 & m=0 \\ 0 & m \neq 0 \end{cases}$
- $p(t)$, Nyquist pulse



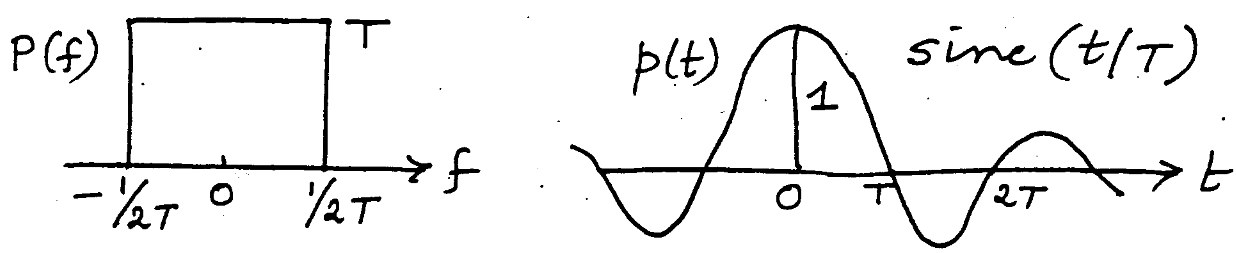
- Equivalently

$$p(t) \sum \delta(t+kT) = \delta(t)$$

$$P(f) * \frac{1}{T} \sum \delta\left(f+\frac{k}{T}\right) = 1$$

$$\sum_k P\left(f+\frac{k}{T}\right) = T$$

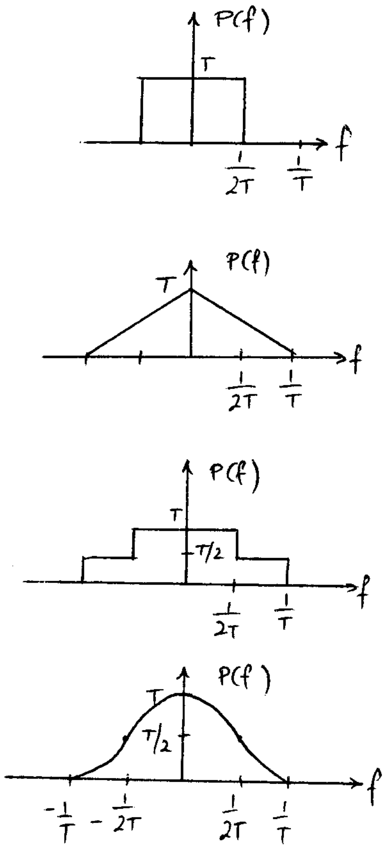
- Minimum bandwidth solution

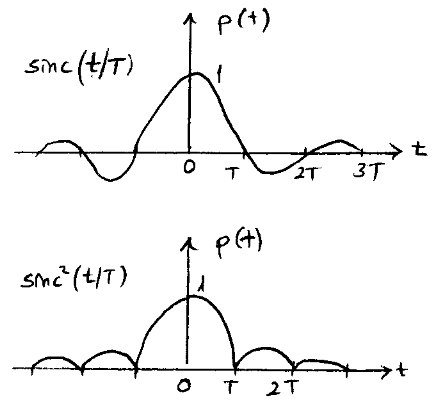


- Minimum bandwidth needed to send independent symbols $\{a_k\}$ without ISI at a signalling rate of R ($=\frac{1}{T}$ baud) is $\underbrace{\frac{R}{2}\left(=\frac{1}{2T}\right)}_{\text{Nyquist bandwidth}}$ Hz.
- Max. symbol packing rate
  $= 2$ baud/Hz

Spectral Efficiency

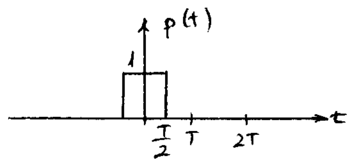
51





* Any $P(f)$ shape odd symmetric wrt the point $\left(\frac{1}{2T}, \frac{T}{2}\right)$ will satisfy the Nyquist criterion.

How about



$$\Sigma P\left(f+\frac{k}{T}\right) \overset{?}{=} T$$

ec 20, Oct 27/99

54

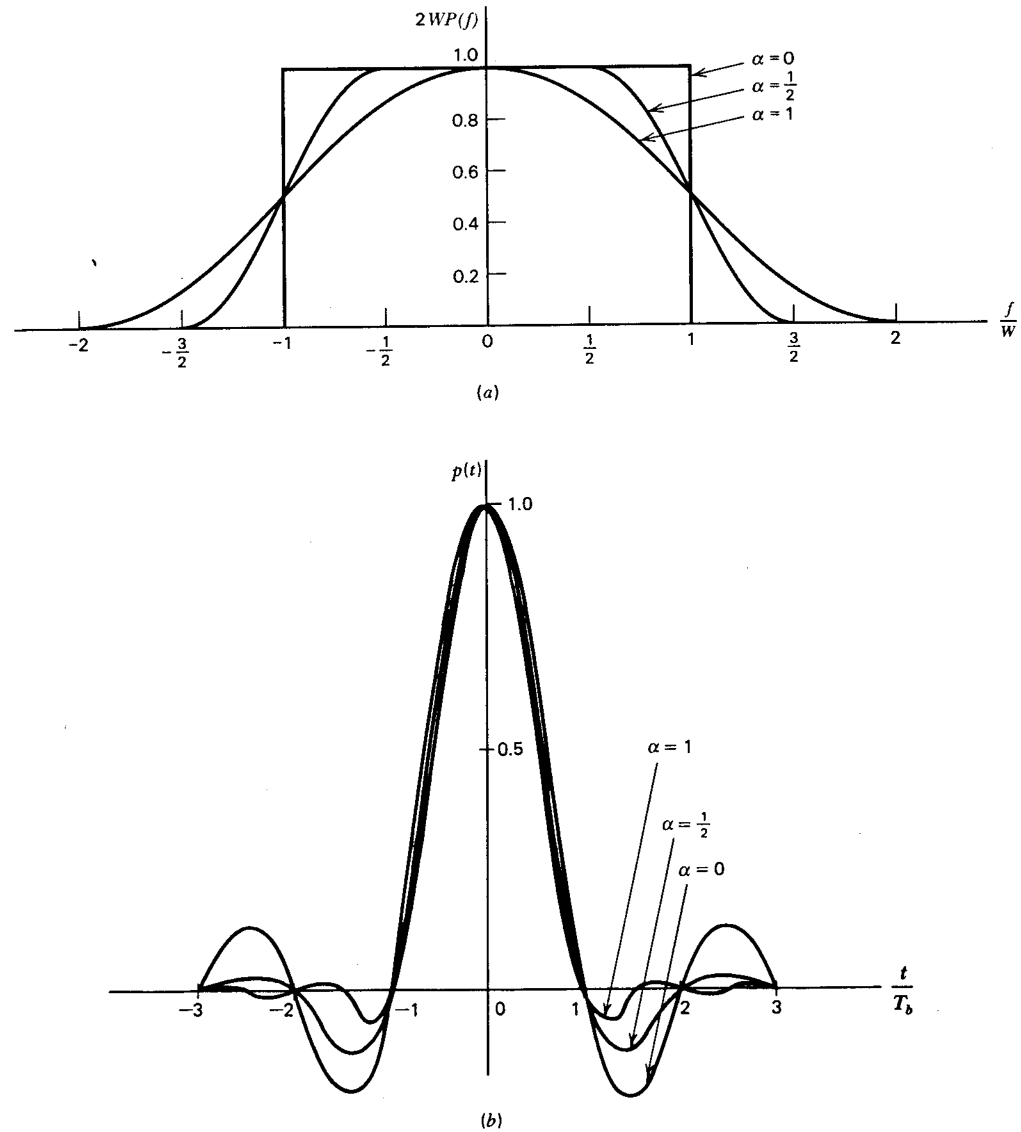
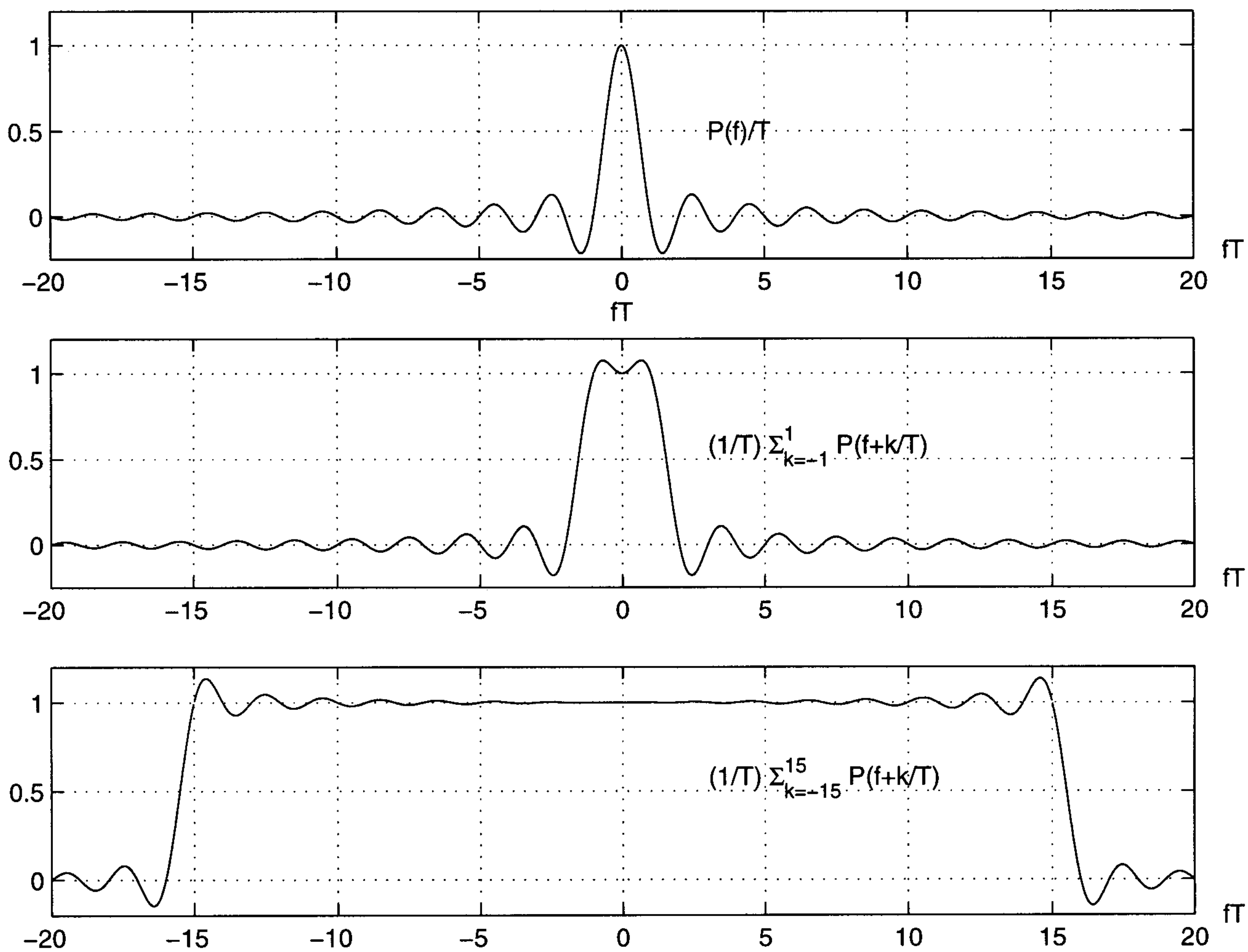
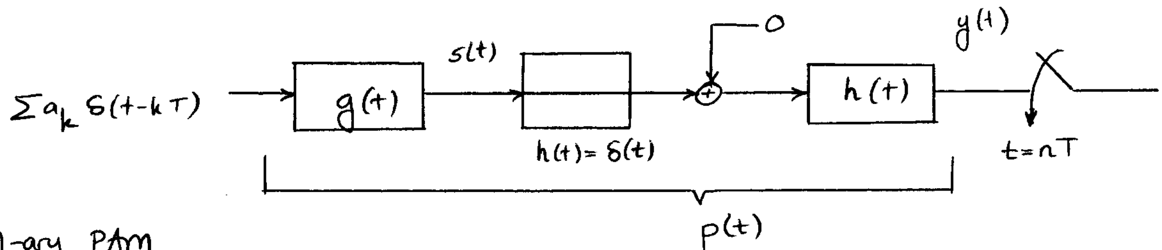


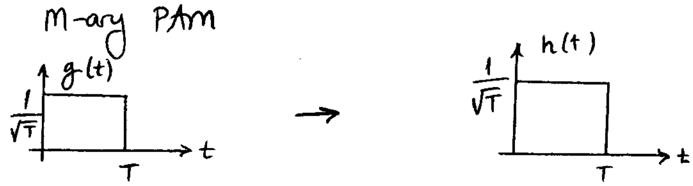
Figure 7.10 Responses for different rolloff factors. (*a*) Frequency response. (*b*) Time response.


P(f)/T
fT
(1/T) Σ$^{1}_{k=-1}$ P(f+k/T)
fT
(1/T) Σ$^{15}_{k=-15}$ P(f+k/T)
fT

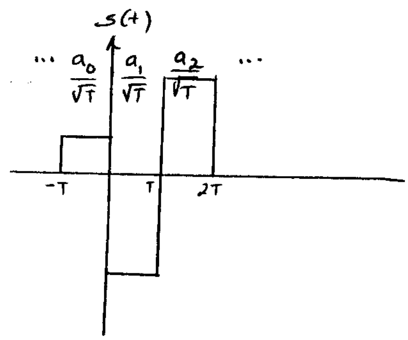
- Is the MF (optimized against noise), satisfy Nyquist (no ISI) criteria, when there is no noise?



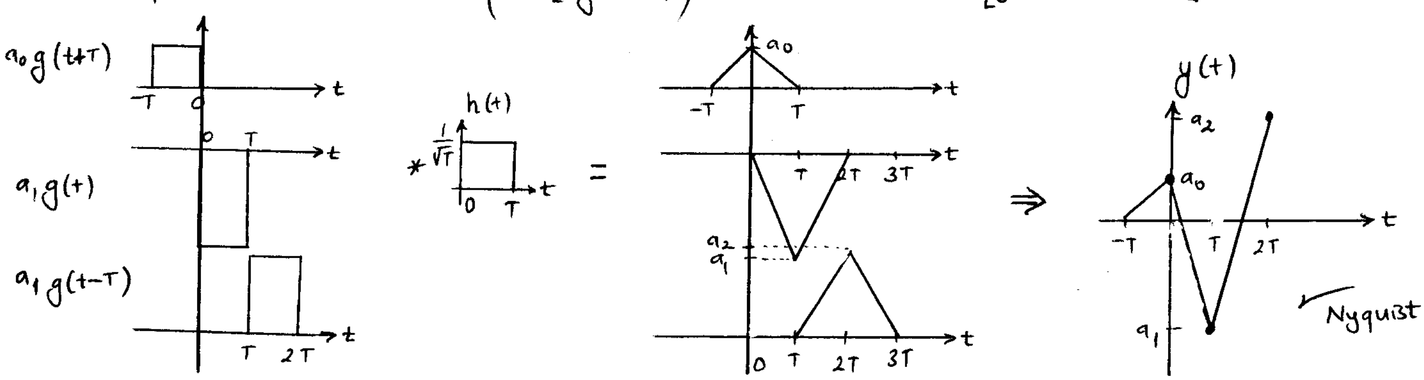
* M-ary PAM

*


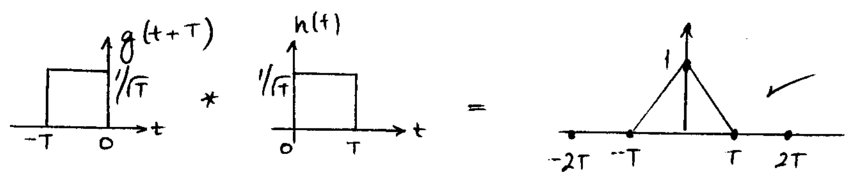
* $s(t) = \sum a_k\, g(t-kT)$



$$p(t) = s(t) * h(t) = \left(\sum a_k\, g(t-kT)\right) * h(t) = \sum a_k \left[g(t-kT) * h(t)\right]$$



Quick check: Is



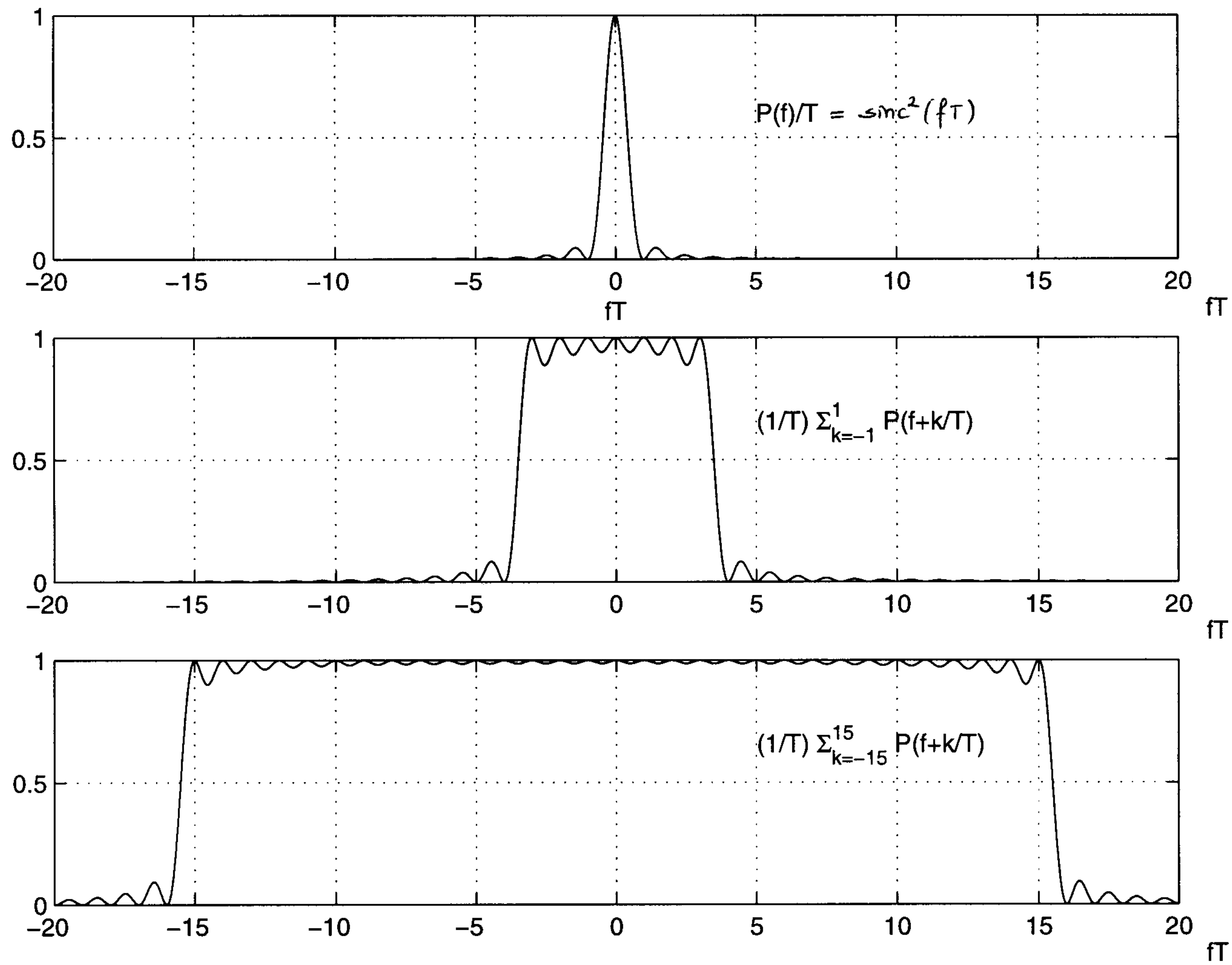
Frequency Domain:

$$P(f) = G(f)\, H(f)$$

$$|P(f)| = |G(f)|^2$$

$$|G(f)| = \sqrt{T}\, \mathrm{sinc}(fT) \;\rightarrow\; |P(f)| = T\, \mathrm{sinc}^2(fT)$$

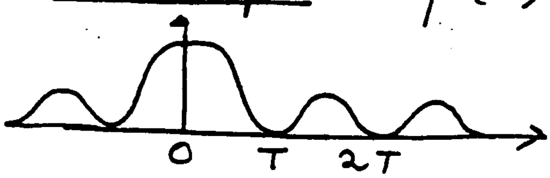
52b

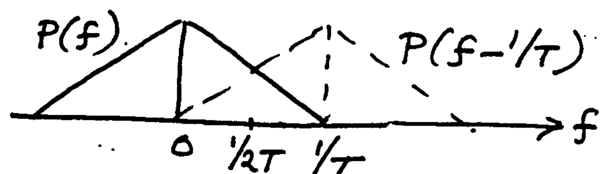


- Transmission at min. BW is impractical due to slow rate of decay of $\text{sinc}(t/T) \Rightarrow$ not robust in presence of timing errors
- Solⁿ : Increase bandwidth

$$p(t) = \text{sinc}(t/T) \cdot x(t)$$ will also satisfy Nyquist's criterion ($x(0)=1$)

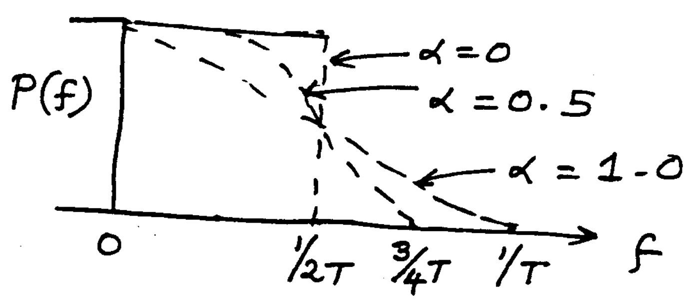
- <u>Example</u>: $p(t) = [\text{sinc}(t/T)]^2$





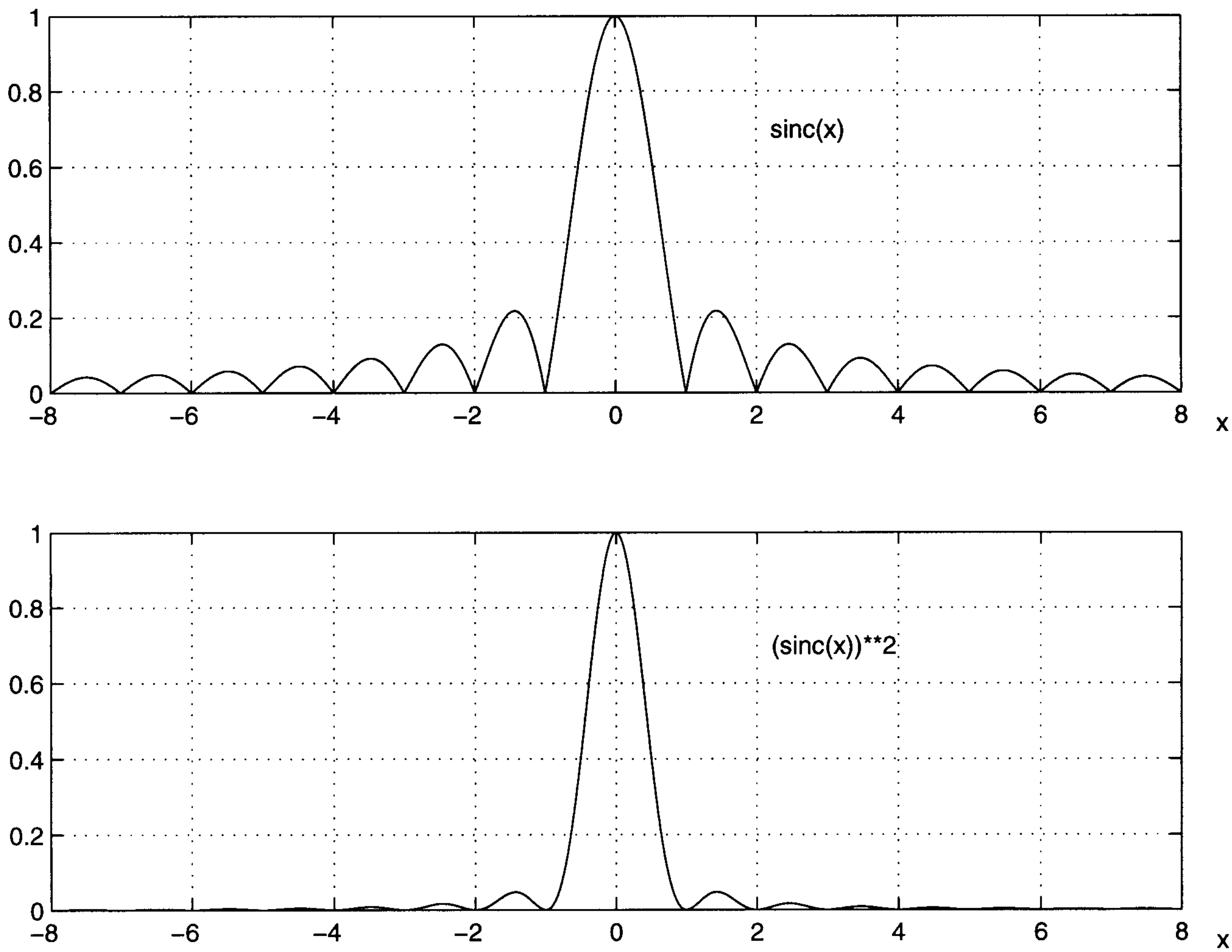
- Popular choice: raised cosine pulses

$$p(t) = \text{sinc}(t/T) \cdot \left(\frac{\cos(\pi \alpha t/T)}{1 - 4\alpha^2 (t/T)^2} \right)$$



$0 \leq \alpha \leq 1$

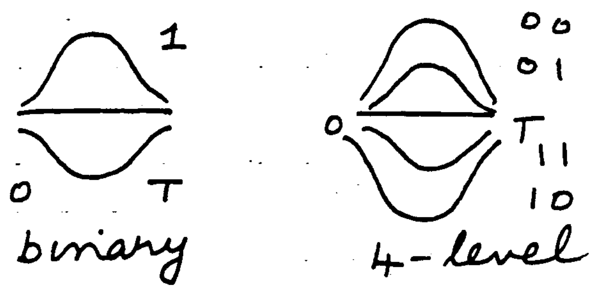
- In practice, 20% to 100% excess bandwidth raised cosine shapes used


1
0.8
0.6
0.4
0.2
0
−8
−6
−4
−2
0
2
4
6
8
x
sinc(x)
(sinc(x))**2

Example :

a) Data rate of 4 level sequence

$R = 2400$ bits/sec

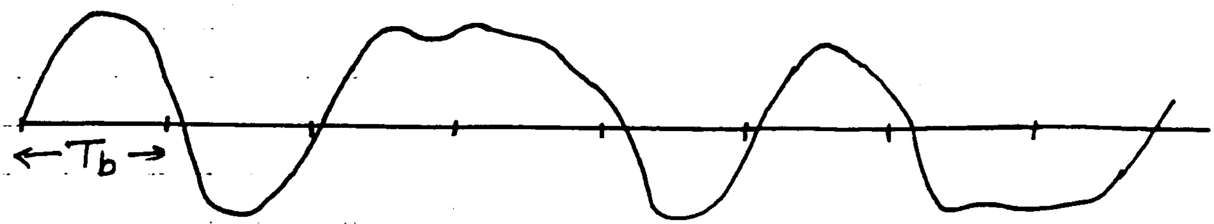


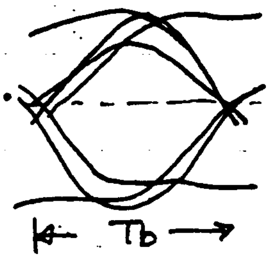
If $P(f) = 100\%$ excess bandwidth raised cosine, what is the minimum bandwidth ?

b) If same data sequence is modulated onto a carrier wave (carrier freq = f_c) and if system transfer function (baseband) is same as in a), find min. DSB bandwidth.

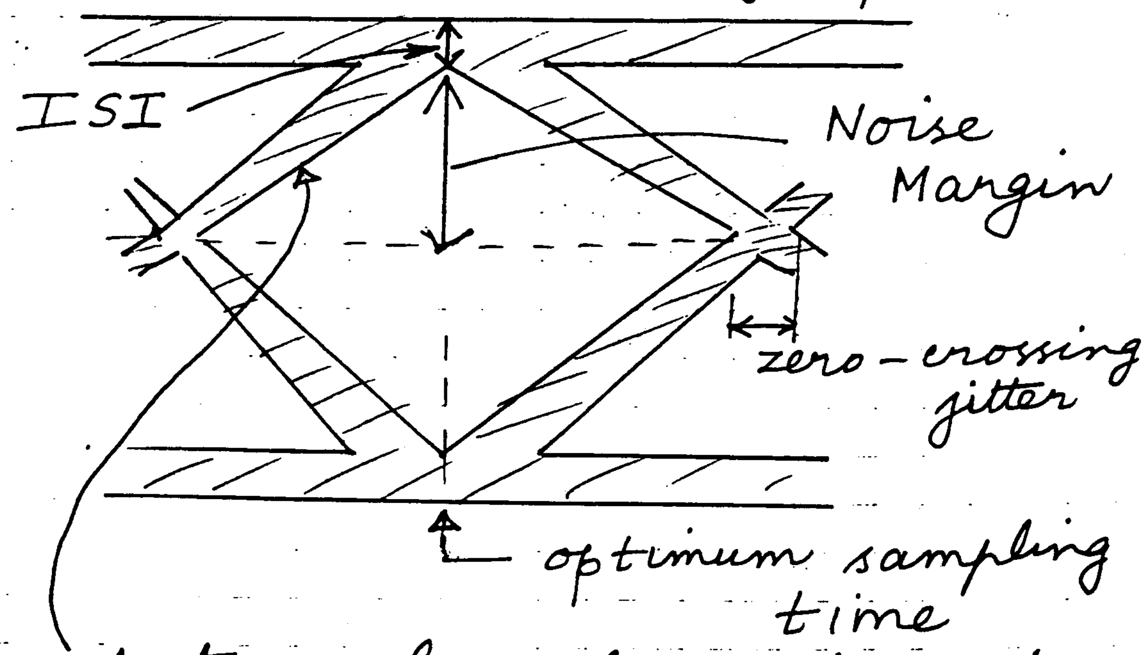
Eye Pattern

- An important experimental display of channel impairments





- noise-free binary signal (with ISI)
- superposition of successive pulses
- Generalized (ideal) eye pattern



rate of slope change indicates sensitivity to timing error

57

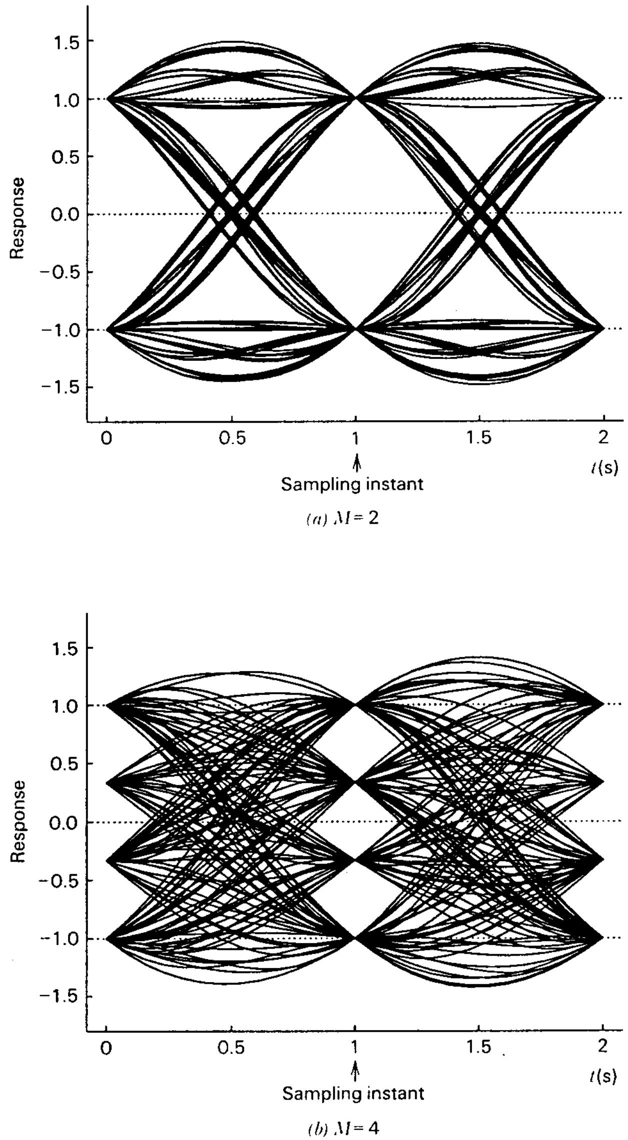


Figure 7.30 Eye diagram of received signal with no bandwidth limitation.

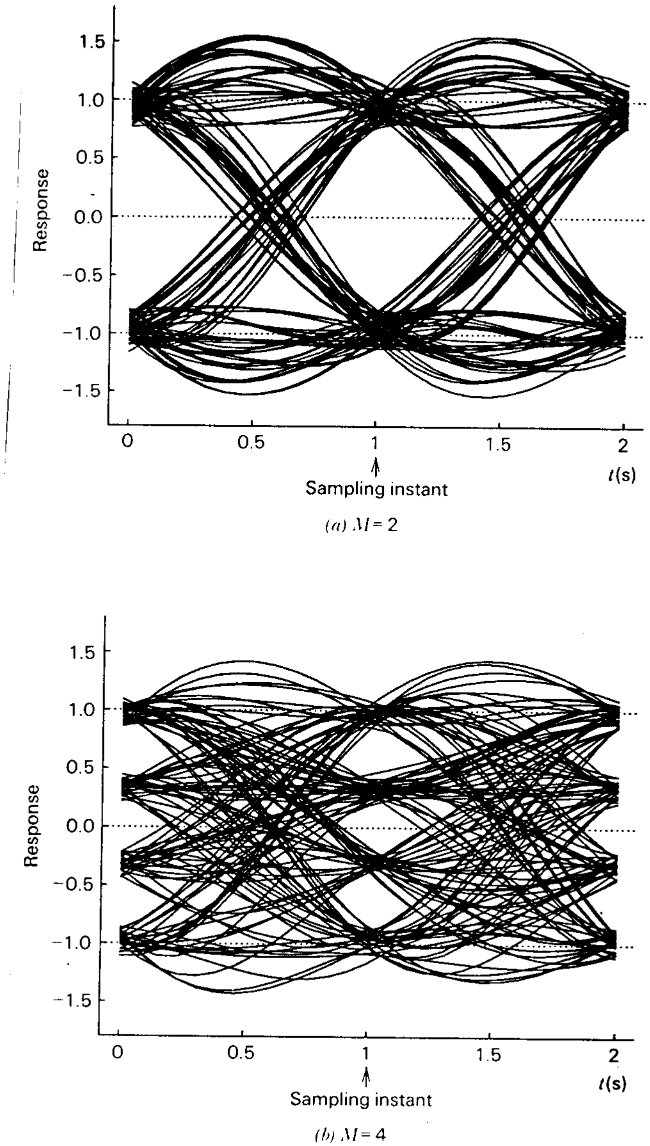
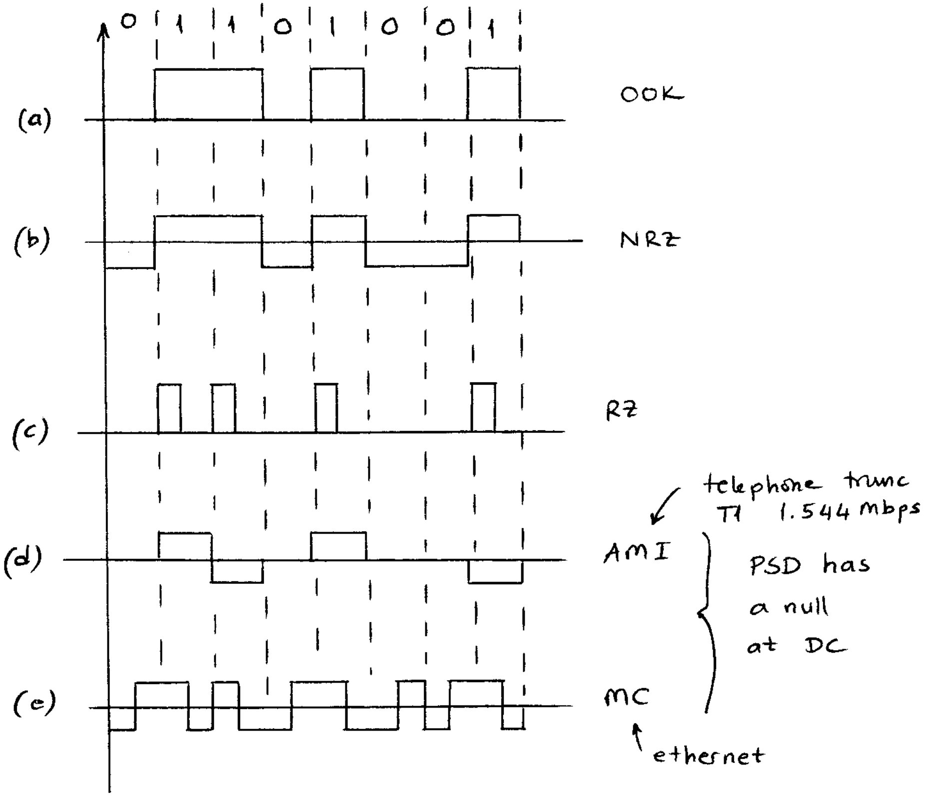
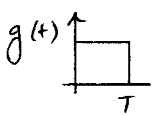


Figure 7.31 Eye diagram of received signal, using a bandwidth-limited channel response.

Line Coding (Formatting) versus Pulse shaping



* Note that the underlying pulse shape is $g(t)$

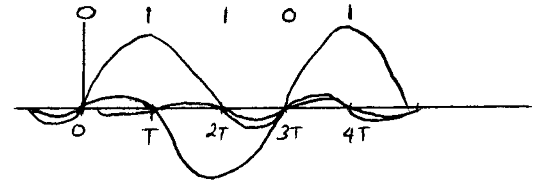


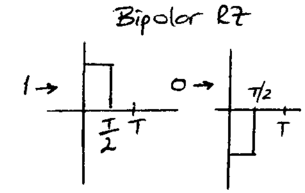
, in the above formatting schemes.

Therefore, (b) is actually NRZ with $g(t)$.

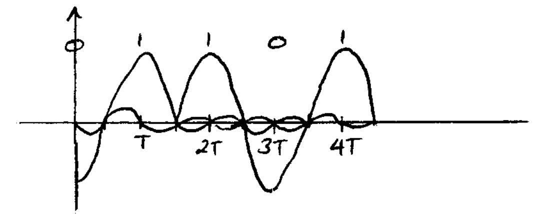
* All of the above formatting (line coding) schemes can be realized with other pulse shapes, such as Nyquist pulses.

- Here is AMI with $p(t) = \text{sinc}(t/T)$ and $a_k \in \{-A, A, 0\}$



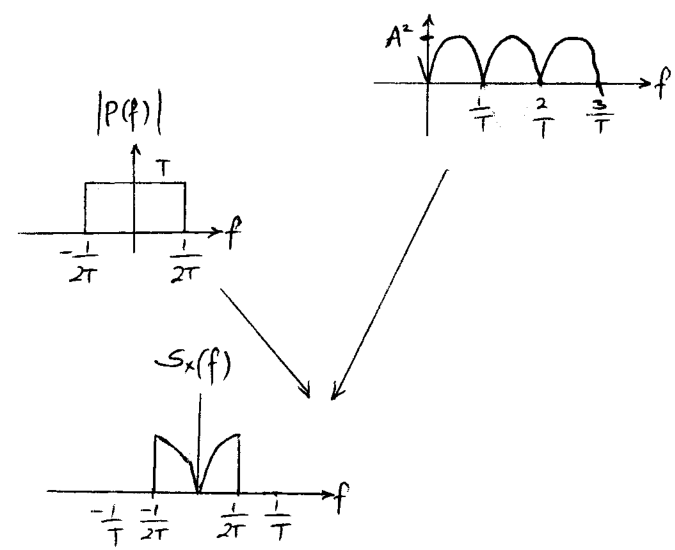


- Bipolar RZ with sinc pulses



- PSD for AMI with $p(t) = \text{sinc}(t/T)$ and $a_k \in \{\pm A, 0\}$

$$S_x(f) = \frac{|P(f)|^2}{T} \underbrace{\sum_{n=-\infty}^{\infty} R_a(n) e^{-j2\pi n f T}}_{A^2 \sin^2(\pi f T)}$$



- Formatting types that yield a null at the 0-frequency are important for baseband transmission; because cables do not pass DC power.

- For bandpass transmission, having a null at the center frequency is not an issue.

- Design the pulse shape according to available BW. If BW is not an issue, rectangular pulses can be used. Otherwise, use Nyquist pulses; most likely raised cosine or square-root raised cosine pulses.

- In a baseband system with raised cosine pulses and with rate R_s ($T_s = \frac{1}{R_s}$),

$$BW = \frac{1+\alpha}{2T_s}$$

raised-cosine roll off factor $(0 < \alpha < 1)$

In a DSB modulated system

$$BW = 2 \times \frac{1+\alpha}{2T_s}$$

Note that $R_b = R_s \times \log_2 M$

— Chapter 7 finished —

Nov 1 / 22

Digital Passband Transmission (ch 8)

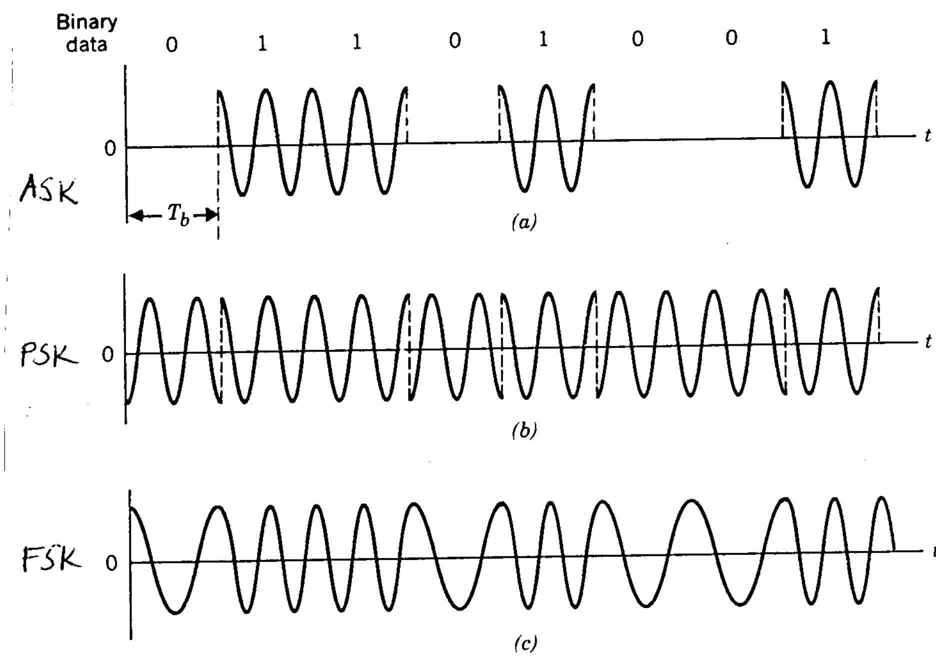
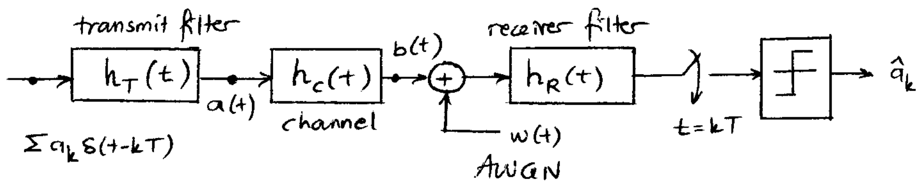


Figure 8.2 The three basic forms of signaling binary information. (*a*) Amplitude-shift keying. (*b*) Phase-shift keying. (*c*) Frequency-shift keying with continuous phase.

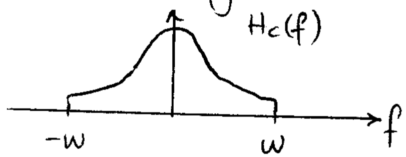
- PSK and FSK: constant envelope (desirable in channels with amplitude distortion)

60a

DESIGN EXAMPLE



- Given the following channel,



with $W = 3$ MHz, can you design a system with $R_b = 15$ mbps ?

- First, ignore the background noise
- The channel causes 2 problems
  - (a) Bandlimited
  - (b) Not flat → pulse shaping

$$S_B(f) = |H_c(f)|^2 S_A(f)$$

- For no-ISI, max transmission rate is determined from the BW of the channel.

Design $h_T(t)$ and $h_R(t)$ such that

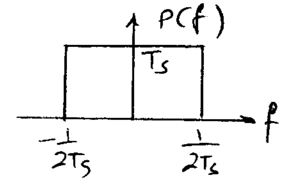
$p(t) = h_T(t) * h_c(t) * h_R(t)$ is Nyquist.

Time Domain check: $p(t) = \begin{cases} 1, & t=0 \\ 0, & t = nT_s \end{cases}$

Frequency Domain check: $\sum_{k=-\infty}^{\infty} P(f + \frac{k}{T_s}) = T_s$

60b

- Min BW solution → sinc pulses (equivalent)

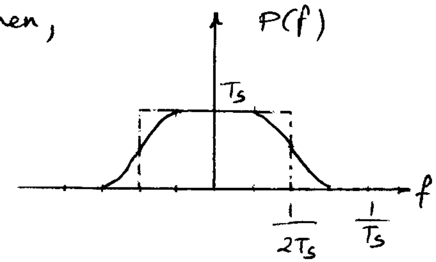


$\rightarrow \frac{1}{2T_s} = W \rightarrow R_s = 2W$ sym/sec

- But due to practical limitations (such as, imperfect sampling times), use raised-cosine pulses.

Say $\alpha = 0.5$ (50% excess BW)

Then,



$\frac{1}{2T_s}(1+\alpha) = W \rightarrow R_s = \frac{2W}{1+\alpha} = 4$ Msym/sec

- Since $R_b = 15$ mbps, binary PAM is not sufficient, use M-ary signalling.
- 1 symbol $= \log_2 M$ bits

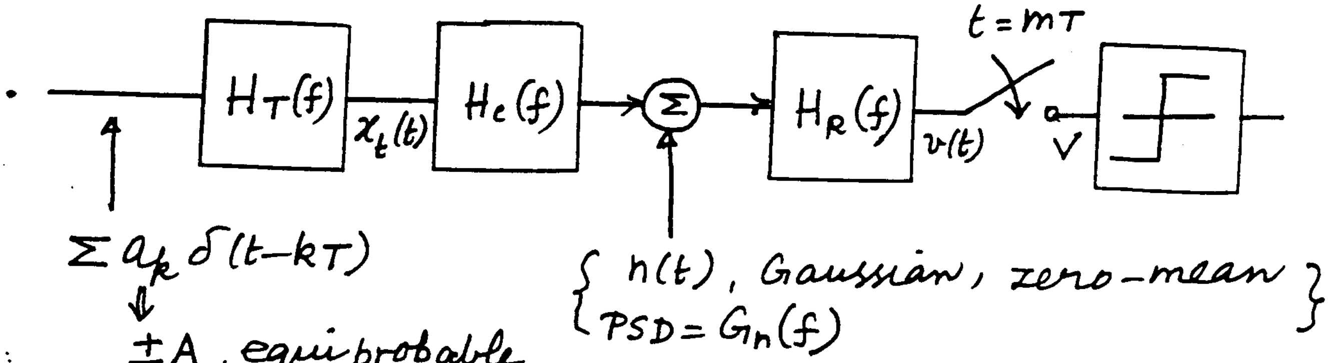
$\rightarrow 4 \times \log_2 M \geq 15 \quad \rightarrow$ use 16-ary PAM ✓

- Question: With $W = 3$ MHz, can you design a system with $R_b = 15$ mbps, and $BER = \gamma_0$?

- BER threshold will determine the maximum M-ary signalling. (For a given E_b, P_e will reduce as M increases)

8. OPTIMUM TERMINAL FILTERS

- System design for ISI + noise

- 

- $$v(t) = \Sigma a_k \, p(t - kT) + \underbrace{n_o(t)}_{n(t) * h_R(t)}$$

$$p(t) = h_T(t) * h_c(t) * h_R(t)$$

- $$V = v(mT) = a_m + \underbrace{N}_{n_o(mT)}$$

if $P(f)$ satisfies Nyquist's criterion

- $$Pe = P(V > 0 / a_m = -A)$$
$$= P(N > A)$$

- N is zero-mean, $var(N) = \sigma^2 = \int_{-\infty}^{\infty} G_n(f) \, |H_R(f)|^2 df$

- $$Pe = \int_A^{\infty} \frac{e^{-u^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}} = \frac{1}{2} \operatorname{erfc}\left(\frac{A}{\sqrt{2}\,\sigma}\right)$$

- $\min Pe \equiv \min \left(\frac{\sigma}{A}\right)^2$ subject to
  i) $P(f)$ is Nyquist
  ii) Transmitted energy fixed at E_T.

- $$E_T = E(a_k^2) \int_{-\infty}^{\infty} h_T^2(t-kT)\, dt$$
$$= A^2 \int_{-\infty}^{\infty} |H_T(f)|^2 df = A^2 \, \| H_T(f) \|^2$$

- $$E_T = A^2 \left\| \frac{|P(f)|}{|H_c(f)||H_R(f)|} \right\|^2$$

- $$\frac{\sigma^2}{A^2} = \frac{\| \sqrt{G_n(f)}\ |H_R(f)|\ \|^2}{E_T} \left\| \frac{|P(f)|}{|H_c(f)|\ |H_R(f)|} \right\|^2$$

$$= \frac{\|\,|X(f)|\,\|^2\ \|\,|Y(f)|\,\|^2}{E_T}$$

$$\underset{\substack{\uparrow \\ \text{Schwarz's} \\ \text{Inequality}}}{\geq} \frac{(|X(f)|\ ,\ |Y(f)|\ )^2}{E_T}$$

with equality iff $|X(f)| = \alpha^2\, |Y(f)|$

or $$|H_R(f)| = \frac{\alpha \sqrt{|P(f)|}}{G_n^{1/4}(f)\ \sqrt{|H_c(f)|}} \qquad (\alpha,\ \text{arbitrary})$$

- Using the min. value for σ/A,

$$P_{e,\min} = \frac{1}{2}\, \mathrm{erfc}\left[\frac{\sqrt{E_T}}{\sqrt{2}} \left\{ \int_{-\infty}^{\infty} \frac{\sqrt{G_n(f)}\ |P(f)|}{|H_c(f)|}\, df \right\}^{-1} \right]$$

- For white noise with $PSD = \frac{N_0}{2}$

and $H_c(f) = 1$ (ideal channel)

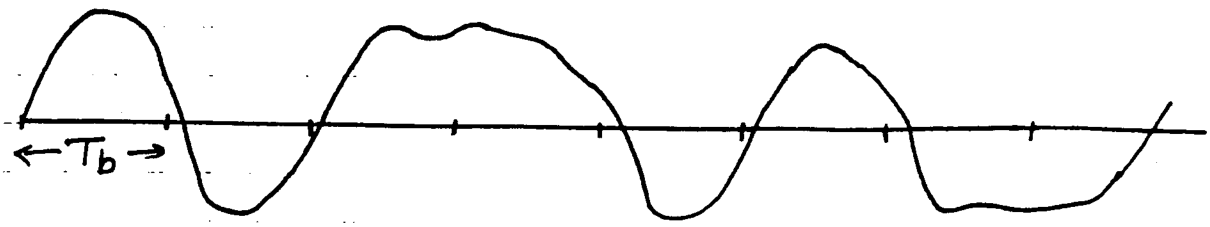
$$|H_T(f)|_{opt} = |H_R(f)|_{opt} = \sqrt{|P(f)|}$$

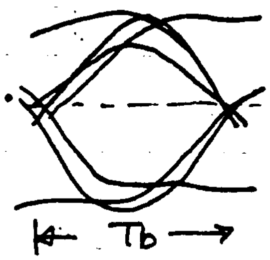
- Nyquist filtering + Matched Filtering $\Rightarrow$ both satisfied

$$P_e = \frac{1}{2}\, \mathrm{erfc} \sqrt{\frac{E_T}{N_0}}$$

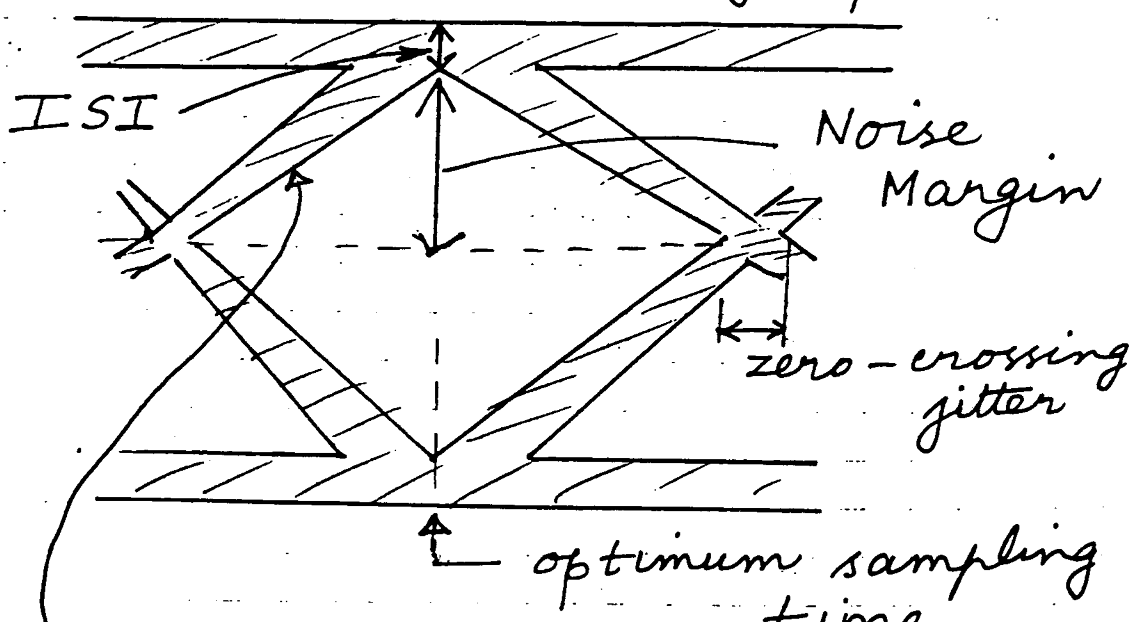
Eye Pattern

- An important experimental display of channel impairments





- noise-free binary signal (with ISI)
- superposition of successive pulses
- Generalized (ideal) eye pattern



rate of slope change indicates sensitivity to timing error

11. ADAPTIVE EQUALIZATION

- The optimum MLSE and the suboptimal LE or DFE assume that the channel characteristics are known.

- But in almost all applications
  (i) the channel is unknown a priori,
  and/or (ii) the channel is time-varying.

- → Automatically adjust the equalizer coefficients $\{c_j\}$ to optimize the performance index and to adaptively compansate for the time variations in the channel.

- Adaptive algorithms → * rate of convergence
  * computational complexity

11.1 Adaptive Linear Equalizer

11.1.1 The zero-Forcing Algorithm

- choose $\{c_j\}$ to minimize $D(\underline{c})$
- In general, there is no simple computational algorithm
- But, for the special case of $D_o = \frac{1}{|f_0|}\sum_{n=1}^{L}|f_n| < 1$ (eye pattern open), choose $\{c_j\}$ such that
  $q_0 = 1$ and $q_n = 0 \ \forall \ 1 \le |n| \le K$.

- For this case,

$$E(\varepsilon_k I_{k-j}^*) = 0, \qquad j = -K, \dots, K$$

- But $E(\varepsilon_k I_{k-j}^*)$ are unknown (since the channel is unknown).

- Transmit a known training sequence $\{I_k\}$ $\rightarrow$ estimate the cross-correlation $E(\varepsilon_k I_{k-j}^*)$ by substituting time averages for the ensemble averages.

- Adjust $\{c_j\}$ through the following recursive formula: (zero-forcing algorithm)

$$\underbrace{c_j^{(k+1)}}_{\text{value of the } j^{th} \text{ coefficient at } t=(k+1)T} = c_j^{(k)} + \underbrace{\Delta}_{\text{scale factor}} \varepsilon_k I_{k-j}^*, \quad j=-K,\dots,K \quad \Big\} \text{ "training mode"}$$

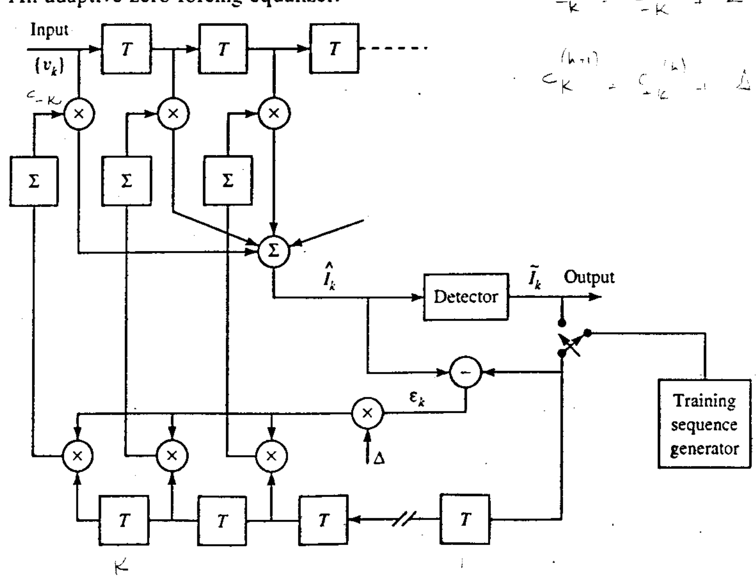
- After the training period (after the coefficients have converged to their optimal values), switch to the "decision-directed mode". At this point, the detector output is reliable enough to continue the adaptation process.

$$c_j^{(k+1)} = c_j^{(k)} + \Delta \underbrace{\tilde{\varepsilon}_k}_{\tilde{I}_k - \hat{I}_k} \tilde{I}_{k-j}^*$$

FIGURE 11-1-1 An adaptive zero-forcing equalizer.

$c_{-K}^{(k+1)} = c_{-K}^{(k)} + \Delta \tilde{\varepsilon}_k \tilde{I}_{k+K}^*$

$c_{K}^{(k+1)} = c_{K}^{(k)} + \Delta \tilde{\varepsilon}_k \tilde{I}_{k-K}^*$



1.1.2 The LMS Algorithm

- MSE criteria

$$\underline{\Gamma}\,\underline{C} = \underline{\xi}$$

$\underline{\Gamma}$: $(2K+1)\times(2K+1)$ covariance matrix

$\underline{C}$: $(2K+1)\times 1$ eq. coefficients vector

$\underline{\xi}$: $(2K+1)\times 1$ channel filter coefficients

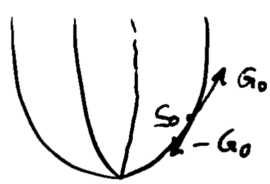
- $\underline{C}_{opt} = \underline{\Gamma}^{-1}\underline{\xi}$ (use an algorithm like Levinson-Durbin)

- Or, matrix inversion can be performed iteratively

use <u>steepest descent</u> (<u>gradient</u>) algo (the simplest iterative algo)

(i) choose an arbitrary $\underline{C}_0$.

(ii)



quadratic MSE surface in the $(2K+1)$-dimensional space

(iii)

$$\underline{C}_{k+1} = \underline{C}_k - \Delta \underline{G}_k, \quad k = 0, 1, 2, \ldots$$

where

$$\underline{G}_k = \frac{1}{2}\frac{dJ}{d\underline{C}_k} = \underline{\Gamma}\,\underline{C}_k - \underline{\xi} = -E\left(\varepsilon_k \underline{V}_k^*\right)$$

$\underline{V}_k$: vector of received signal samples

(iv) continue until $\underline{G}_k = \underline{0}$

- In general, $J_{min}(K)$ cannot be achived with finite # of steps in the steepest descent algo. However, can be closely approached.

- $\underline{G}_k$ depends on $\underline{\Gamma}$ and $\underline{\xi}$. $\underline{\Gamma}$ and $\underline{\xi}$ depend on $\{f_j\}_{j=1}^{L}$.

But we do not know $\{f_j\}$!

- use estimates of the gradient

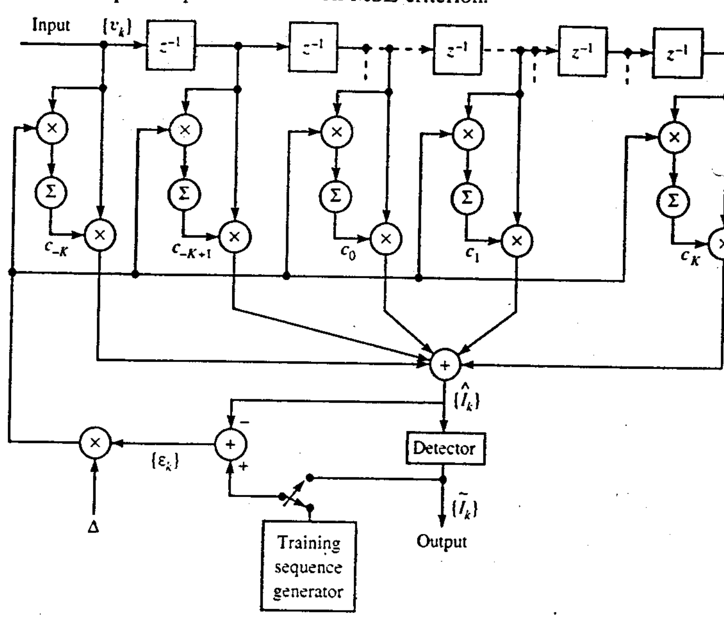
$$\hat{\underline{C}}_{k+1} = \hat{\underline{C}}_k - \Delta \hat{\underline{G}}_k$$

where $\hat{\underline{G}}_k = -\varepsilon_k \underline{V}_k^*$

$$\therefore \quad \hat{\underline{C}}_{k+1} = \hat{\underline{C}}_k + \Delta \varepsilon_k \underline{V}_k^* \quad \ldots$$

LMS (least-mean-square) algo
(stochastic gradient)

FIGURE 11-1-2 Linear adaptive equalizer based on MSE criterion.



$$\hat{C}_0^{(k+1)} = \hat{C}_0^{(k)} + \Delta \varepsilon^{(k)} V_0^*$$

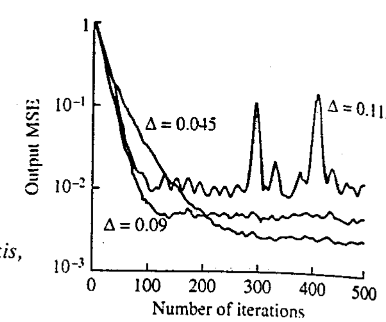


FIGURE 11-1-4 Initial convergence characteristics of the LMS algorithm with different step sizes. [*From* Digital Signal Processing, *by J. G. Proakis and D. G. Manolakis, 1988, Macmillan Publishing Company. Reprinted with permission of the publisher.*]

11.2 Adaptive Decision-Feedback Equalizer

- MSE criteria

<u>Steepest-descent</u> algorithm <u>(gradient)</u>:

$$\underline{C}_{k+1} = \underline{C}_k + \Delta E(\varepsilon_k \underline{V}_k^*)$$

- MSE is minimized when the cross-correlation vector $E(\varepsilon_k \underline{V}_k^*) = \underline{0}$ as $k \to \infty$.

- Actual cross-correlation values are not known
  → use estimates

⇒ <u>LMS</u> algo : $\hat{\underline{C}}_{k+1} = \hat{\underline{C}}_k + \Delta \varepsilon_k \underline{V}_k^*$ (training mode)
(<u>stochastic gradient</u>)

$$\tilde{\underline{C}}_{k+1} = \tilde{\underline{C}}_k + \Delta \underbrace{\tilde{\varepsilon}_k}_{\tilde{I}_k - \hat{I}_k} \underline{V}_k^*$$ (decision-directed mode) (adaptive mode)

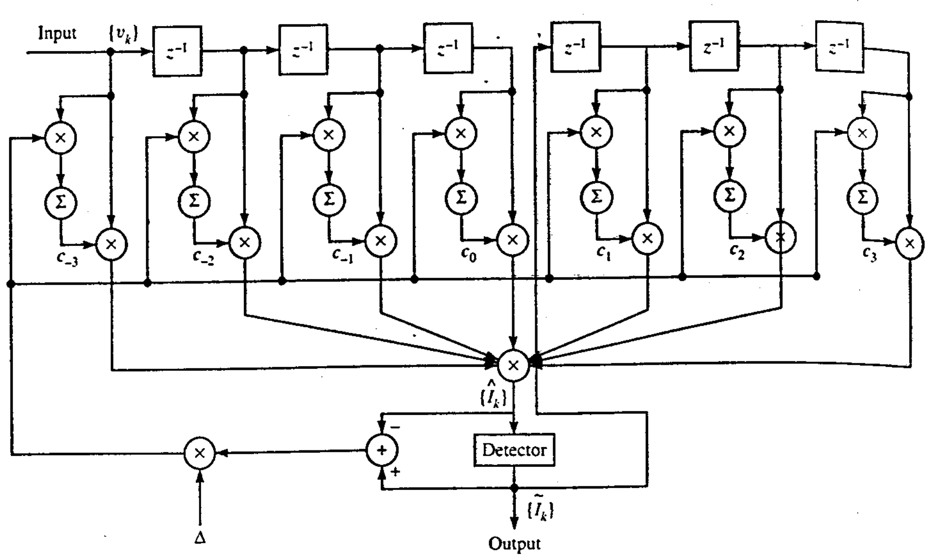
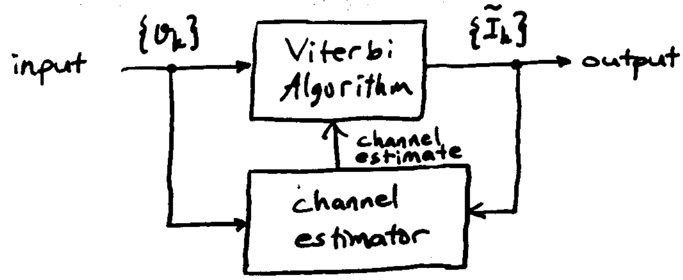


FIGURE 11-2-1 Decision-feedback equalizer.

11.3 Adaptive Channel Estimation for ML Sequence Detector



$$\hat{\underline{f}}_{k+1} = \hat{\underline{f}}_k + \Delta \varepsilon_k \tilde{\underline{I}}_k^* \quad \text{(decision-directed mode)}$$

11.4 Recursive Least-Squares Algorithms for Adaptive Equalization

- In a stochastic gradient algorithm (LMS), the true gradient vector is approximated by an estimate obtained directly from the data.

- LMS: computationally simple. But, converges slowly because their is only single adjustable parameter (Δ).

- In $\underline{\Gamma}\, \underline{c} = \underline{\xi}$ type of sistem, the speed of convergence is related to the eigenvalue spread of $\underline{\Gamma}$. If $\underline{\Gamma}$ is $N \times N$, it has N eigenvalues: $\lambda_1, \cdots, \lambda_N$. If $\lambda_{max} / \lambda_{min} \gg 1$

  $\rightarrow$ convergence slow.

- Since $\underline{\Gamma}$ has N eigenvalues, develop an iterative algorithm with N parameters (one for each eigenvalue). The optimum selection of these parameters yields a faster convergence rate.

  $\rightarrow$ Recursive Least-Squares algo
  (Kalman algo)

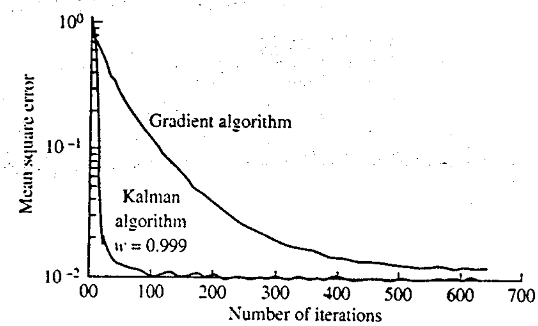


FIGURE 11-4-1 Comparison of convergence rate for the Kalman and gradient algorithms.

- Price paid for Kalman algorithm → complexity

11.5 Self-Recovering (Blind) Equalization

- initial adjustments without a training sequence

(i) stochastic gradient (LMS) → godard algo
(constant modulus algo) CMA

(ii) 2nd and higher or statistics
(of the received signal to estimate the channel characteristics)

(iii) ML criteria