Problem 6.15 :

The received signal-plus-noise vector at the output of the matched filter may be represented as (see (5-2-63) for example) :

$$r_n = \sqrt{\mathcal{E}_s} e^{j(\theta_n - \phi)} + N_n$$

where $\theta_n = 0, \pi/2, \pi, 3\pi/2$ for QPSK, and ϕ is the carrier phase. By raising r_n to the fourth power and neglecting all products of noise terms, we obtain :

$$\begin{aligned} r_n^4 &\approx \left(\sqrt{\mathcal{E}_s}\right)^4 e^{j4(\theta_n - \phi)} + 4\left(\sqrt{\mathcal{E}_s}\right)^3 N_n \\ &\approx \left(\sqrt{\mathcal{E}_s}\right)^3 \left[\sqrt{\mathcal{E}_s} e^{-j4\phi} + 4N_n\right] \end{aligned}$$

If the estimate is formed by averaging the received vectors $\{r_n^4\}$ over K signal intervals, we have the resultant vector $U = K\sqrt{\mathcal{E}_s} e^{-j\phi} + 4\sum_{n=1}^{K} N_n$. Let $\phi_4 \equiv 4\phi$. Then, the estimate of ϕ_4 is :

$$\hat{\phi}_4 = -\tan^{-1} \frac{Im(U)}{Re(U)}$$

N_n is a complex-valued Gaussian noise component with zero mean and variance $\sigma^2 = N_0/2$. Hence, the pdf of $\hat{\phi}_4$ is given by (5-2-55) where :

$$\gamma_s = \frac{\left(K\sqrt{\mathcal{E}_s}\right)^2}{16\left(2K\sigma^2\right)} = \frac{K^2\mathcal{E}_s}{16KN_0} = \frac{K\mathcal{E}_s}{16N_0}$$

To a first approximation, the variance of the estimate is :

$$\sigma^2_{\hat{\phi}_4} \approx \frac{1}{\gamma_s} = \frac{16}{K\mathcal{E}_s/N_0}$$

Problem 6.16 :

The PDF of the carrier phase error ϕ_e, is given by :

$$p(\phi_e) = \frac{1}{\sqrt{2\pi}\sigma_\phi} e^{-\frac{\phi_e^2}{2\sigma_\phi^2}}$$

Thus the average probability of error is :

$$\begin{aligned} \bar{P}_2 &= \int_{-\infty}^{\infty} P_2(\phi_e) p(\phi_e) d\phi_e \\ &= \int_{-\infty}^{\infty} Q\left[\sqrt{\frac{2\mathcal{E}_b}{N_0}\cos^2\phi_e}\right] p(\phi_e) d\phi_e \\ &= \frac{1}{2\pi\sigma_\phi} \int_{-\infty}^{\infty} \int_{\sqrt{\frac{2\mathcal{E}_b}{N_0}\cos^2\phi_e}}^{\infty} \exp\left[-\frac{1}{2}\left(x^2 + \frac{\phi_e^2}{\sigma_\phi^2}\right)\right] dx d\phi_e \end{aligned}$$

Problem 6.17:

The log-likelihood function of the symbol timing may be expressed in terms of the equivalent low-pass signals as

$$\begin{aligned} \Lambda_L(\tau) &= \Re\left[\tfrac{1}{N_0}\int_{T_0} r(t)s_l{}^*(t;\tau)dt\right] \\ &= \Re\left[\tfrac{1}{N_0}\int_{T_0} r(t)\sum_n I_n{}^* g^*(t-nT-\tau)dt\right] \\ &= \Re\left[\tfrac{1}{N_0}\sum_n I_n{}^* y_n(\tau)\right] \end{aligned}$$

where $y_n(\tau)=\int_{T_0} r(t)g^*(t-nT-\tau)dt$.
A necessary condition for $\hat{\tau}$ to be the ML estimate of τ is

$$\begin{aligned} \tfrac{d\Lambda_L(\tau)}{\tau} &= 0 \Rightarrow \\ \tfrac{d}{d\tau}\left[\sum_n I_n{}^* y_n(\tau)+\sum_n I_n y_n{}^*(\tau)\right] &= 0 \Rightarrow \\ \sum_n I_n{}^* \tfrac{d}{d\tau}y_n(\tau)+\sum_n I_n \tfrac{d}{d\tau}y_n{}^*(\tau) &= 0 \end{aligned}$$

If we express $y_n(\tau)$ into its real and imaginary parts : $y_n(\tau)=a_n(\tau)+jb_n(\tau)$, the above expression simplifies to the following condition for the ML estimate of the timing $\hat{\tau}$

$$\sum_n \Re[I_n]\frac{d}{d\tau}a_n(\tau)+\sum_n \Im[I_n]\frac{d}{d\tau}b_n(\tau)=0$$

Problem 6.18:

We follow the exact same steps of the derivation found in Sec. 6.4. For a PAM signal $I_n{}^*=I_n$ and $J_n=0$. Since the pulse $g(t)$ is real, it follows that $B(\tau)$ in expression (6.4-6) is zero, therefore (6.4-7) can be rewritten as

$$\Lambda_L(\phi,\tau)=A(\tau)\cos\phi$$

where

$$A(\tau)=\frac{1}{N_0}\sum I_n y_n(\tau)$$

Then the necessary conditions for the estimates of ϕ and τ to be the ML estimates (6.4-8) and (6.4-9) give

$$\hat{\phi}_{ML}=0$$

and

$$\sum_n I_n\frac{d}{d\tau}[y_n(\tau)]_{\tau=\hat{\tau}_{ML}}=0$$

Problem 9.15 :

The SNR at the detector is :

$$\frac{\mathcal{E}_b}{N_0} = \frac{P_b T}{N_0} = \frac{P_b(1+\beta)}{N_0 W} = 30 \text{ dB}$$

Since it is desired to expand the bandwidth by a factor of $\frac{10}{3}$ while maintaining the same SNR, the received power P_b should increase by the same factor. Thus the additional power needed is

$$P_a = 10\log_{10}\frac{10}{3} = 5.2288 \text{ dB}$$

Hence, the required transmitted power is :

$$P_S = -3 + 5.2288 = 2.2288 \text{ dBW}$$

Problem 9.16 :

The pulse $x(t)$ having the raised cosine spectrum given by (9-2-26/27) is :

$$x(t) = \text{sinc}(t/T)\frac{\cos(\pi\beta t/T)}{1-4\beta^2t^2/T^2}$$

The function $\text{sinc}(t/T)$ is 1 when $t=0$ and 0 when $t=nT$. Therefore, the Nyquist criterion will be satisfied as long as the function $g(t)$ is :

$$g(t) = \frac{\cos(\pi\beta t/T)}{1-4\beta^2t^2/T^2} = \begin{cases} 1 & t=0 \\ \text{bounded} & t\neq 0 \end{cases}$$

The function $g(t)$ needs to be checked only for those values of t such that $4\beta^2t^2/T^2 = 1$ or $\beta t = \frac{T}{2}$. However :

$$\lim_{\beta t\to\frac{T}{2}} \frac{\cos(\pi\beta t/T)}{1-4\beta^2t^2/T^2} = \lim_{x\to 1}\frac{\cos(\frac{\pi}{2}x)}{1-x}$$

and by using L'Hospital's rule :

$$\lim_{x\to 1}\frac{\cos(\frac{\pi}{2}x)}{1-x} = \lim_{x\to 1}\frac{\pi}{2}\sin(\frac{\pi}{2}x) = \frac{\pi}{2} < \infty$$

Hence :

$$x(nT) = \begin{cases} 1 & n=0 \\ 0 & n\neq 0 \end{cases}$$

meaning that the pulse $x(t)$ satisfies the Nyquist criterion.

Problem 9.17 :

Substituting the expression of $X_{rc}(f)$ given by (8.2.22) in the desired integral, we obtain :

$$\int_{-\infty}^{\infty} X_{rc}(f)df \;=\; \int_{-\frac{1+\beta}{2T}}^{-\frac{1-\beta}{2T}} \frac{T}{2}\left[1+\cos\frac{\pi T}{\beta}(-f-\frac{1-\beta}{2T})\right]df + \int_{-\frac{1-\beta}{2T}}^{\frac{1-\beta}{2T}} T df$$

$$
\begin{aligned}
& + \int_{\frac{1-\beta}{2T}}^{\frac{1+\beta}{2T}} \frac{T}{2} \left[1 + \cos \frac{\pi T}{\beta} (f - \frac{1-\beta}{2T}) \right] df \\
= \; & \int_{-\frac{1+\beta}{2T}}^{-\frac{1-\beta}{2T}} \frac{T}{2} df + T \left(\frac{1-\beta}{T} \right) + \int_{\frac{1-\beta}{2T}}^{\frac{1+\beta}{2T}} \frac{T}{2} df \\
& + \int_{-\frac{1+\beta}{2T}}^{-\frac{1-\beta}{2T}} \cos \frac{\pi T}{\beta} (f + \frac{1-\beta}{2T}) df + \int_{\frac{1-\beta}{2T}}^{\frac{1+\beta}{2T}} \cos \frac{\pi T}{\beta} (f - \frac{1-\beta}{2T}) df \\
= \; & 1 + \int_{-\frac{\beta}{T}}^{0} \cos \frac{\pi T}{\beta} x dx + \int_{0}^{\frac{\beta}{T}} \cos \frac{\pi T}{\beta} x dx \\
= \; & 1 + \int_{-\frac{\beta}{T}}^{\frac{\beta}{T}} \cos \frac{\pi T}{\beta} x dx = 1 + 0 = 1
\end{aligned}
$$

Problem 9.19 :

The bandwidth of the channel is :

$$W = 3000 - 300 = 2700 \text{ Hz}$$

Since the minimum transmission bandwidth required for bandpass signaling is R, where R is the rate of transmission, we conclude that the maximum value of the symbol rate for the given channel is $R_{\max} = 2700$. If an M-ary PAM modulation is used for transmission, then in order to achieve a bit-rate of 9600 bps, with maximum rate of $R_{\max}$, the minimum size of the constellation is $M = 2^k = 16$. In this case, the symbol rate is :

$$R = \frac{9600}{k} = 2400 \text{ symbols/sec}$$

and the symbol interval $T = \frac{1}{R} = \frac{1}{2400}$ sec. The roll-off factor β of the raised cosine pulse used for transmission is is determined by noting that $1200(1 + \beta) = 1350$, and hence, $\beta = 0.125$. Therefore, the squared root raised cosine pulse can have a roll-off of $\beta = 0.125$.

Problem 9.23 :

The roll-off factor β is related to the bandwidth by the expression $\frac{1+\beta}{T} = 2W$, or equivalently $R(1+\beta) = 2W$. The following table shows the symbol rate for the various values of the excess bandwidth and for $W = 1500$ Hz.

β	.25	.33	.50	.67	.75	1.00
R	2400	2256	2000	1796	1714	1500

The above results were obtained with the assumption that double-sideband PAM is employed, so the available lowpass bandwidth will be from $-W = \frac{3000}{2}$ to W Hz. If single-sideband transmission is used, then the spectral efficiency is doubled, and the above symbol rates R are doubled.

Problem 9.24 :

The following table shows the precoded sequence, the transmitted amplitude levels, the received signal levels and the decoded sequence, when the data sequence 10010110010 modulates a duobinary transmitting filter.

Data seq. D_n:		1	0	0	1	0	1	1	0	0	1	0
Precoded seq. P_n:	0	1	1	1	0	0	1	0	0	0	1	1
Transmitted seq. I_n:	-1	1	1	1	-1	-1	1	-1	-1	-1	1	1
Received seq. B_n:		0	2	2	0	-2	0	0	-2	-2	0	2
Decoded seq. D_n:		1	0	0	1	0	1	1	0	0	1	0

Problem 9.25 :

The following table shows the precoded sequence, the transmitted amplitude levels, the received signal levels and the decoded sequence, when the data sequence 10010110010 modulates a modified duobinary transmitting filter.

Data seq. D_n:			1	0	0	1	0	1	1	0	0	1	0
Precoded seq. P_n:	0	0	1	0	1	1	1	0	0	0	0	1	0
Transmitted seq. I_n:	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	1	-1
Received seq. B_n:			2	0	0	2	0	-2	-2	0	0	2	0
Decoded seq. D_n:			1	0	0	1	0	1	1	0	0	1	0

Problem 10.1 :

Suppose that $a_m = +1$ is the transmitted signal. Then the probability of error will be :

$$\begin{aligned} P_{e|1} &= P\left(y_m < 0 | a_m = +1\right) \\ &= P\left(1 + n_m + i_m < 0\right) \\ &= \frac{1}{4}P\left(1/2 + n_m < 0\right) + \frac{1}{4}P\left(3/2 + n_m < 0\right) + \frac{1}{2}P\left(1 + n_m < 0\right) \\ &= \frac{1}{4}Q\left[\frac{1}{2\sigma_n}\right] + \frac{1}{4}Q\left[\frac{3}{2\sigma_n}\right] + \frac{1}{2}Q\left[\frac{1}{\sigma_n}\right] \end{aligned}$$

Due to the symmetry of the intersymbol interference, the probability of error, when $a_m = -1$ is transmitted, is the same. Thus, the above result is the average probability of error.

Problem 10.10 :

(a) The equivalent discrete-time impulse response of the channel is :

$$h(t) = \sum_{n=-1}^{1} h_n \delta(t - nT) = 0.3\delta(t + T) + 0.9\delta(t) + 0.3\delta(t - T)$$

If by $\{c_n\}$ we denote the coefficients of the FIR equalizer, then the equalized signal is :

$$q_m = \sum_{n=-1}^{1} c_n h_{m-n}$$

which in matrix notation is written as :

$$\begin{pmatrix} 0.9 & 0.3 & 0. \\ 0.3 & 0.9 & 0.3 \\ 0. & 0.3 & 0.9 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

The coefficients of the zero-force equalizer can be found by solving the previous matrix equation. Thus,

$$\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} -0.4762 \\ 1.4286 \\ -0.4762 \end{pmatrix}$$

(b) The values of q_m for $m = \pm 2, \pm 3$ are given by

$$\begin{aligned} q_2 &= \sum_{n=-1}^{1} c_n h_{2-n} = c_1 h_1 = -0.1429 \\ q_{-2} &= \sum_{n=-1}^{1} c_n h_{-2-n} = c_{-1} h_{-1} = -0.1429 \\ q_3 &= \sum_{n=-1}^{1} c_n h_{3-n} = 0 \\ q_{-3} &= \sum_{n=-1}^{1} c_n h_{-3-n} = 0 \end{aligned}$$

Problem 10.11 :

(a) The output of the zero-force equalizer is :

$$q_m = \sum_{n=-1}^{1} c_n x_{m_n}$$

With $q_0 = 1$ and $q_m = 0$ for $m \neq 0$, we obtain the system :

$$\begin{pmatrix} 1.0 & 0.1 & -0.5 \\ -0.2 & 1.0 & 0.1 \\ 0.05 & -0.2 & 1.0 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Solving the previous system in terms of the equalizer's coefficients, we obtain :

$$\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0.000 \\ 0.980 \\ 0.196 \end{pmatrix}$$

(b) The output of the equalizer is :

$$q_m = \begin{cases} 0 & m \leq -4 \\ c_{-1}x_{-2} = 0 & m = -3 \\ c_{-1}x_{-1} + c_0 x_{-2} = -0.49 & m = -2 \\ 0 & m = -1 \\ 1 & m = 0 \\ 0 & m = 1 \\ c_0 x_2 + x_1 c_1 = 0.0098 & m = 2 \\ c_1 x_2 = 0.0098 & m = 3 \\ 0 & m \geq 4 \end{cases}$$

Hence, the residual ISI sequence is

$$\text{residual ISI} = \{\ldots, 0, -0.49, 0, 0, 0, 0.0098, 0.0098, 0, \ldots\}$$

and its span is 6 symbols.

Problem 10.23 :

(a)

$$F(z) = 0.8 - 0.6z^{-1} \Rightarrow$$
$$X(z) \equiv F(z)F^*(z^{-1}) = (0.8 - 0.6z^{-1})(0.8 - 0.6z) = 1 - 0.48z^{-1} - 0.48z$$

Thus, $x_0 = 1,\ x_{-1} = x_1 = -0.48$.

(b)

$$\frac{1}{T}\sum_{n=-\infty}^{\infty}\left|H\left(\omega + \frac{2\pi n}{T}\right)\right|^2 = X\left(e^{j\omega T}\right) = 1 - 0.48e^{-j\omega T} - 0.48e^{j\omega T} = 1 - 0.96\cos\omega T$$

(c) For the linear equalizer base on the mean-square-error criterion we have :

$$\begin{aligned} J_{\min} &= \tfrac{T}{2\pi}\int_{-\pi/T}^{\pi/T}\tfrac{N_0}{1+N_0-0.96\cos\omega T}d\omega \\ &= \tfrac{1}{2\pi}\int_{-\pi}^{\pi}\tfrac{N_0}{1+N_0-0.96\cos\theta}d\theta \\ &= \tfrac{1}{2\pi}\left(\tfrac{N_0}{1+N_0}\right)\int_{-\pi}^{\pi}\tfrac{1}{1-a\cos\theta}d\theta,\ a = \tfrac{0.96}{1+N_0} \end{aligned}$$

But :

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{1-a\cos\theta}d\theta = \frac{1}{\sqrt{1-a^2}},\ a^2 < 1$$

Therefore :

$$J_{\min} = \frac{N_0}{1+N_0}\frac{1}{\sqrt{1-\left(\frac{0.96}{1+N_0}\right)^2}} = \frac{N_0}{\sqrt{(1+N_0)^2-(0.96)^2}}$$

(d) For the decision-feedback equalizer :

$$J_{\min} = \frac{2N_0}{1+N_0+\sqrt{(1+N_0)^2-(0.96)^2}}$$

which follows from the result in example 10.3.1. Note that for $N_0 << 1$,

$$J_{\min} \approx \frac{2N_0}{1+\sqrt{1-(0.96)^2}} \approx 1.56N_0$$

In contrast, for the linear equalizer we have :

$$J_{\min} \approx \frac{N_0}{\sqrt{1-(0.96)^2}} \approx 3.57N_0$$

(d) The metrics are

$$(y_1 - 0.8I_1)^2\,,\ i=1 \ \text{ and } \sum_i (y_i - 0.8I_i + 0.6I_{i-1})^2\,,\ i \geq 2$$

$$\begin{aligned}
\mu_1\,(I_1 = 3) &= [0.5 - 3*0.8]^2 = 3.61\\
\mu_1\,(I_1 = 1) &= [0.5 - 1*0.8]^2 = 0.09\\
\mu_1\,(I_1 = -1) &= [0.5 + 1*0.8]^2 = 1.69\\
\mu_1\,(I_1 = -3) &= [0.5 + 3*0.8]^2 = 8.41
\end{aligned}$$

$$\begin{aligned}
&\mu_2\,(I_2 = 3, I_1 = 3) = \mu_1(3) + [2 - 2.4 + 3*0.6]^2 = 5.57\\
&\mu_2\,(3,1) = \mu_1(1) + [2 - 2.4 + 1*0.6]^2 = 0.13\\
&\mu_2\,(3,-1) = \mu_1(-1) + [2 - 2.4 - 1*0.6]^2 = 6.53\\
&\mu_2\,(3,-3) = \mu_1(-3) + [2 - 2.4 - 3*0.6]^2 = 13.25
\end{aligned}$$

$$\begin{aligned}
&\mu_2\,(1,3) = \mu_1(3) + [2 - 0.8 + 3*0.6]^2 = 12.61\\
&\mu_2\,(1,1) = \mu_1(1) + [2 - 0.8 + 1*0.6]^2 = 3.33\\
&\mu_2\,(1,-1) = \mu_1(-1) + [2 - 0.8 - 1*0.6]^2 = 2.05\\
&\mu_2\,(1,-3) = \mu_1(-3) + [2 - 0.8 - 3*0.6]^2 = 8.77
\end{aligned}$$

$$\begin{aligned}
&\mu_2\,(-1,3) = \mu_1(3) + [2 + 0.8 + 3*0.6]^2 = 24.77\\
&\mu_2\,(-1,1) = \mu_1(1) + [2 + 0.8 + 1*0.6]^2 = 11.65\\
&\mu_2\,(-1,-1) = \mu_1(-1) + [2 + 0.8 - 1*0.6]^2 = 6.53\\
&\mu_2\,(-1,-3) = \mu_1(-3) + [2 + 0.8 - 3*0.6]^2 = 9.41
\end{aligned}$$

$$\begin{aligned}
&\mu_2\,(-3,3) = \mu_1(3) + [2 + 2.4 + 3*0.6]^2 = 42.05\\
&\mu_2\,(-3,1) = \mu_1(1) + [2 + 2.4 + 1*0.6]^2 = 25.09\\
&\mu_2\,(-3,-1) = \mu_1(-1) + [2 + 2.4 - 1*0.6]^2 = 16.13\\
&\mu_2\,(-3,-3) = \mu_1(-3) + [2 + 2.4 - 3*0.6]^2 = 15.17
\end{aligned}$$

The four surviving paths at this stage are $\min_{I_1}\left[\mu_2(x, I_1)\right],\ x = 3, 1, -1, -3$ or :

$$\begin{aligned}
&I_2 = 3, I_1 = 1 \text{ with metric } \mu_2(3,1) = 0.13\\
&I_2 = 1, I_1 = -1 \text{ with metric } \mu_2(1,-1) = 2.05\\
&I_2 = -1, I_1 = -1 \text{ with metric } \mu_2(-1,-1) = 6.53\\
&I_2 = -3, I_1 = -3 \text{ with metric } \mu_2(-3,-3) = 15.17
\end{aligned}$$

Now we compute the metrics for the next stage :

$$\begin{aligned}
&\mu_3\,(I_3 = 3, I_2 = 3, I_1 = 1) = \mu_2(3,1) + [-1 - 2.4 + 1.8]^2 = 2.69\\
&\mu_3\,(3,1,-1) = \mu_2(1,-1) + [-1 - 2.4 + 0.6]^2 = 9.89\\
&\mu_3\,(3,-1,-1) = \mu_2(-1,-1) + [-1 - 2.4 - 0.6]^2 = 22.53\\
&\mu_3\,(3,-3,-3) = \mu_2(-3,-3) + [-1 - 2.4 - 1.8]^2 = 42.21
\end{aligned}$$

$$
\begin{aligned}
\mu_3\,(1,3,1) &= \mu_2(3,1) + [-1-0.8+1.8]^2 = 0.13\\
\mu_3\,(1,1,-1) &= \mu_2(1,-1) + [-1-0.8+0.6]^2 = 7.81\\
\mu_3\,(1,-1,-1) &= \mu_2(-1,-1) + [-1-0.8-0.6]^2 = 12.29\\
\mu_3\,(1,-3,-3) &= \mu_2(-3,-3) + [-1-0.8-1.8]^2 = 28.13
\end{aligned}
$$

$$
\begin{aligned}
\mu_3\,(-1,3,1) &= \mu_2(3,1) + [-1+0.8+1.8]^2 = 2.69\\
\mu_3\,(-1,1,-1) &= \mu_2(1,-1) + [-1+0.8+0.6]^2 = 2.69\\
\mu_3\,(-1,-1,-1) &= \mu_2(-1,-1) + [-1+0.8-0.6]^2 = 7.17\\
\mu_3\,(-1,-3,-3) &= \mu_2(-3,-3) + [-1+0.8-1.8]^2 = 19.17
\end{aligned}
$$

$$
\begin{aligned}
\mu_3\,(-3,3,1) &= \mu_2(3,1) + [-1+2.4+1.8]^2 = 10.37\\
\mu_3\,(-3,1,-1) &= \mu_2(1,-1) + [-1+2.4+0.6]^2 = 2.69\\
\mu_3\,(-3,-1,-1) &= \mu_2(-1,-1) + [-1+2.4-0.6]^2 = 7.17\\
\mu_3\,(-3,-3,-3) &= \mu_2(-3,-3) + [-1+2.4-1.8]^2 = 15.33
\end{aligned}
$$

The four surviving sequences at this stage are $\min_{I_2,I_1}[\mu_3(x,I_2,I_1)]\,,\ x = 3,1,-1,-3$ or :

$$
\begin{aligned}
&I_3 = 3, I_2 = 3, I_1 = 1 \text{ with metric } \mu_3(3,3,1) = 2.69\\
&I_3 = 1, I_2 = 3, I_1 = 1 \text{ with metric } \mu_3(1,3,1) = 0.13\\
&I_3 = -1, I_2 = 3, I_1 = 1 \text{ with metric } \mu_3(-1,3,1) = 2.69\\
&I_3 = -3, I_2 = 1, I_1 = -1 \text{ with metric } \mu_3(-3,1,-1) = 2.69
\end{aligned}
$$

(e) For the channel, $\delta^2_{\min} = 1$ and hence :

$$
P_4 = 8Q\left(\sqrt{\frac{6}{15}\gamma_{av}}\right)
$$