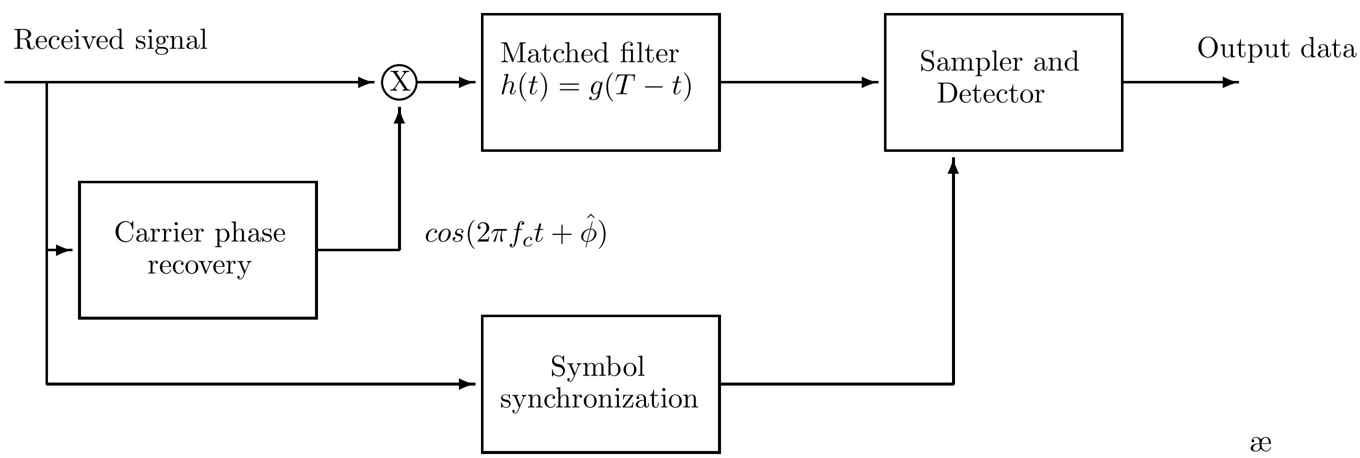
Problem 6.2 :

A block diagram of a binary PSK receiver that employs match filtering is given in the following figure :



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As we note, the received signal is, first, multiplied with $cos(2\pi f_c t + \hat{\phi})$ and then fed the matched filter. This allows us to have the filter matched to the baseband pulse $g(t)$ and not to the passband signal.

If we want to have the filter matched to the passband signal, then the carrier phase estimate is fed into the matched filter, which should have an impulse response:

$$\begin{aligned} h(t) &= s(T-t) = g(T-t)cos(2\pi f_c(T-t)+\hat{\phi}) \\ &= g(T-t)[cos(2\pi f_c T)cos(-2\pi f_c t+\hat{\phi}) + sin(2\pi f_c T)sin(-2\pi f_c t+\hat{\phi}) \\ &= g(T-t)cos(-2\pi f_c t+\hat{\phi}) = g(T-t)cos(2\pi f_c t-\hat{\phi}) \end{aligned}$$

where we have assumed that $f_c T$ is an integer so that : $cos(2\pi f_c T) = 1,\ sin(2\pi f_c T) = 0$. As we note, in this case the impulse response of the filter should change according to the carrier phase estimate, something that is difficult to implement in practise. Hence, the initial realization (shown in the figure) is preferable.

Problem 6.8 :

An on-off keying signal is represented as :

$$\begin{array}{ll} s_1(t) = A\cos(2\pi f_c t + \phi_c), & 0 \le t \le T \text{ (binary 1)} \\ s_2(t) = 0, & 0 \le t \le T \text{ (binary 0)} \end{array}$$

Let $r(t)$ be the received signal, that is $r(t) = s(t;\phi_c) + n(t)$ where $s(t;\phi_c)$ is either $s_1(t)$ or $s_2(t)$ and $n(t)$ is white Gaussian noise with variance $\frac{N_0}{2}$. The likelihood function, that is to be maximized with respect to ϕ_c over the inteval $[0, T]$, is proportional to :

$$\Lambda(\phi_c) = \exp\left[-\frac{2}{N_0}\int_0^T [r(t) - s(t;\phi_c)]^2 dt\right]$$

Maximization of $\Lambda(\phi_c)$ is equivalent to the maximization of the log-likelihood function :

$$\begin{aligned}\Lambda_L(\phi_c) &= -\frac{2}{N_0}\int_0^T [r(t)-s(t;\phi_c)]^2 dt \\ &= -\frac{2}{N_0}\int_0^T r^2(t)dt + \frac{4}{N_0}\int_0^T r(t)s(t;\phi_c)dt - \frac{2}{N_0}\int_0^T s^2(t;\phi_c)dt\end{aligned}$$

Since the first term does not involve the parameter of interest ϕ_c and the last term is simply a constant equal to the signal energy of the signal over $[0,T]$ which is independent of the carrier phase, we can carry the maximization over the function :

$$V(\phi_c) = \int_0^T r(t)s(t;\phi_c)dt$$

Note that $s(t;\phi_c)$ can take two different values, $s_1(t)$ and $s_2(t)$, depending on the transmission of a binary 1 or 0. Thus, a more appropriate function to maximize is the average log-likelihood

$$\bar{V}(\phi_c) = \frac{1}{2}\int_0^T r(t)s_1(t)dt + \frac{1}{2}\int_0^T r(t)s_2(t)dt$$

Since $s_2(t) = 0$, the function $\bar{V}(\phi_c)$ takes the form :

$$\bar{V}(\phi_c) = \frac{1}{2}\int_0^T r(t)A\cos(2\pi f_c t + \phi_c)dt$$

Setting the derivative of $\bar{V}(\phi_c)$ with respect to ϕ_c equal to zero, we obtain :

$$\begin{aligned}\frac{\vartheta \bar{V}(\phi_c)}{\vartheta \phi_c} = 0 &= \frac{1}{2}\int_0^T r(t)A\sin(2\pi f_c t + \phi_c)dt \\ &= \cos\phi_c \frac{1}{2}\int_0^T r(t)A\sin(2\pi f_c t)dt + \sin\phi_c \frac{1}{2}\int_0^T r(t)A\cos(2\pi f_c t)dt\end{aligned}$$

Thus, the maximum likelihood estimate of the carrier phase is :

$$\hat{\phi}_{c,ML} = -\arctan\left[\frac{\int_0^T r(t)\sin(2\pi f_c t)dt}{\int_0^T r(t)\cos(2\pi f_c t)dt}\right]$$

Problem 6.9 :

(a) The wavelength λ is :

$$\lambda = \frac{3\times 10^8}{10^9}\ \text{m} = \frac{3}{10}\ \text{m}$$

Hence, the Doppler frequency shift is :

$$f_D = \pm\frac{u}{\lambda} = \pm\frac{100\ \text{Km/hr}}{\frac{3}{10}\ \text{m}} = \pm\frac{100\times 10^3\times 10}{3\times 3600}\text{Hz} = \pm 92.5926\ \text{Hz}$$

The plus sign holds when the vehicle travels towards the transmitter whereas the minus sign holds when the vehicle moves away from the transmitter.

(b) The maximum difference in the Doppler frequency shift, when the vehicle travels at speed 100 km/hr and $f = 1$ GHz, is :

$$\Delta f_{D\max} = 2f_D = 185.1852 \text{ Hz}$$

This should be the bandwith of the Doppler frequency tracking loop.

(c) The maximum Doppler frequency shift is obtained when $f = 1$ GHz $+ 1$ MHz and the vehicle moves towards the transmitter. In this case :

$$\lambda_{\min} = \frac{3 \times 10^8}{10^9 + 10^6} \text{ m } = 0.2997 \text{ m}$$

and therefore :

$$f_{D\max} = \frac{100 \times 10^3}{0.2997 \times 3600} = 92.6853 \text{ Hz}$$

Thus, the Doppler frequency spread is $B_d = 2f_{D\max} = 185.3706$ Hz.

Problem 6.10 :

The maximum likelihood phase estimate given by (6-2-38) is :

$$\hat{\phi}_{ML} = -\tan^{-1} \frac{Im\left[\sum_{n=0}^{K-1} I_n^* y_n\right]}{Re\left[\sum_{n=0}^{K-1} I_n^* y_n\right]}$$

where $y_n = \int_{nT}^{(n+1)T} r(t) g^*(t - nT) dt$. The $Re(y_n)$, $Im(y_n)$ are statistically independent components of y_n. Since $r(t) = e^{-j\phi} \sum_n I_n g(t - nT) + z(t)$ it follows that $y_n = I_n e^{-j\phi} + z_n$, where the pulse energy is normalized to unity. Then :

$$\sum_{n=0}^{K-1} I_n^* y_n = \sum_{n=0}^{K-1} \left[|I_n|^2 e^{-j\phi} + I_n^* z_n \right]$$

Hence :

$$E\left\{ Im\left[\sum_{n=0}^{K-1} \left[|I_n|^2 e^{-j\phi} + I_n^* z_n \right] \right] \right\} = -K \left| \bar{I}_n \right|^2 \sin\phi$$

and

$$E\left\{ Re\left[\sum_{n=0}^{K-1} \left[|I_n|^2 e^{-j\phi} + I_n^* z_n \right] \right] \right\} = -K \left| \bar{I}_n \right|^2 \cos\phi$$

Consequently : $E\left[\hat{\phi}_{ML}\right] = -\tan^{-1}\frac{-\sin\phi}{\cos\phi} = \phi$, and hence, $\hat{\phi}_{ML}$ is an unbiased estimate of the true phase ϕ.

Problem 6.13 :

Assume that the signal $u_m(t)$ is the input to the Costas loop. Then $u_m(t)$ is multiplied by $\cos(2\pi f_c t + \hat{\phi})$ and $\sin(2\pi f_c t + \hat{\phi})$, where $\cos(2\pi f_c t + \hat{\phi})$ is the output of the VCO. Hence :

$$\begin{aligned}
&u_{mc}(t)\\
&= A_m g_T(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \hat{\phi}) - A_m \hat{g}_T(t)\sin(2\pi f_c t)\cos(2\pi f_c t + \hat{\phi})\\
&= \frac{A_m g_T(t)}{2}\left[\cos(2\pi 2 f_c t + \hat{\phi}) + \cos(\hat{\phi})\right] - \frac{A_m \hat{g}_T(t)}{2}\left[\sin(2\pi 2 f_c t + \hat{\phi}) - \sin(\hat{\phi})\right]\\
&u_{ms}(t)\\
&= A_m g_T(t)\cos(2\pi f_c t)\sin(2\pi f_c t + \hat{\phi}) - A_m \hat{g}_T(t)\sin(2\pi f_c t)\sin(2\pi f_c t + \hat{\phi})\\
&= \frac{A_m g_T(t)}{2}\left[\sin(2\pi 2 f_c t + \hat{\phi}) + \sin(\hat{\phi})\right] - \frac{A_m \hat{g}_T(t)}{2}\left[\cos(\hat{\phi}) - \cos(2\pi 2 f_c t + \hat{\phi})\right]
\end{aligned}$$

The lowpass filters of the Costas loop will reject the double frequency components, so that :

$$\begin{aligned}
y_{mc}(t) &= \frac{A_m g_T(t)}{2}\cos(\hat{\phi}) + \frac{A_m \hat{g}_T(t)}{2}\sin(\hat{\phi})\\
y_{ms}(t) &= \frac{A_m g_T(t)}{2}\sin(\hat{\phi}) - \frac{A_m \hat{g}_T(t)}{2}\cos(\hat{\phi})
\end{aligned}$$

Note that when the carrier phase has been extracted correctly, $\hat{\phi} = 0$ and therefore :

$$y_{mc}(t) = \frac{A_m g_T(t)}{2}, \qquad y_{ms}(t) = -\frac{A_m \hat{g}_T(t)}{2}$$

If the second signal, $y_{ms}(t)$ is passed through a Hilbert transformer, then :

$$\hat{y}_{ms}(t) = -\frac{A_m \hat{\hat{g}}_T(t)}{2} = \frac{A_m g_T(t)}{2}$$

and by adding this signal to $y_{mc}(t)$ we obtain the original unmodulated signal.

Problem 6.16 :

The PDF of the carrier phase error ϕ_e, is given by :

$$p(\phi_e) = \frac{1}{\sqrt{2\pi}\sigma_\phi} e^{-\frac{\phi_e^2}{2\sigma_\phi^2}}$$

Thus the average probability of error is :

$$\begin{aligned}
\bar{P}_2 &= \int_{-\infty}^{\infty} P_2(\phi_e) p(\phi_e) d\phi_e \\
&= \int_{-\infty}^{\infty} Q\left[\sqrt{\frac{2\mathcal{E}_b}{N_0}\cos^2\phi_e}\right] p(\phi_e) d\phi_e \\
&= \frac{1}{2\pi\sigma_\phi} \int_{-\infty}^{\infty} \int_{\sqrt{\frac{2\mathcal{E}_b}{N_0}\cos^2\phi_e}}^{\infty} \exp\left[-\frac{1}{2}\left(x^2 + \frac{\phi_e^2}{\sigma_\phi^2}\right)\right] dx d\phi_e
\end{aligned}$$