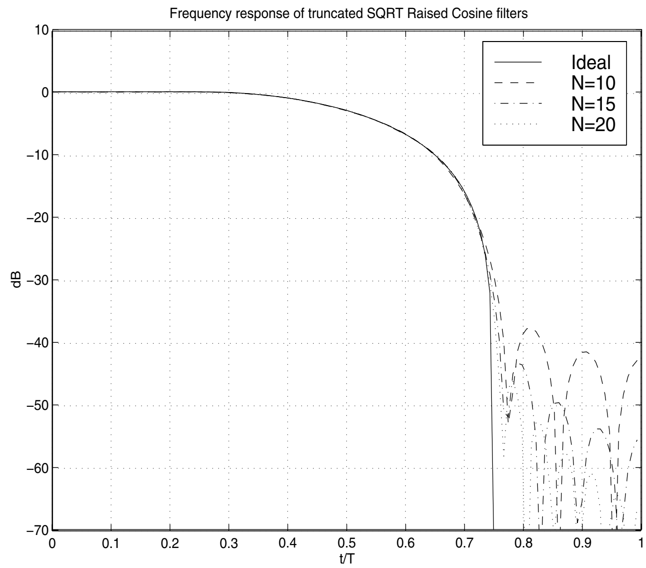
Problem 9.6 :

(a)(b) In order to calculate the frequency response based on the impulse response, we need the values of the impulse response at $t = 0, \pm T/2$, which are not given directly by the expression of Problem 9.5. Using L'Hospital's rule it is straightforward to show that:

$$x(0) = \frac{1}{2} + \frac{2}{\pi}, \quad x(\pm T/2) = \frac{\sqrt{2}}{2}\frac{(2+\pi)}{2\pi}$$

Then, the frequency response of the filters with $N = 10, 15, 20$ compared to the frequency response of the ideal square-root raised cosine filter are depicted in the following figure.



As we see, there is no significant difference in the passband area of the filters, but the realizable, truncated filters do have spectral sidelobes outside their $(1+\beta)/T$ nominal bandwidth. Still, depending on how much residual ISI an application can tolerate, even the $N = 10$ filter appears an acceptable approximation of the ideal (non-realizable) square-root raised cosine filter.

Problem 9.10 :

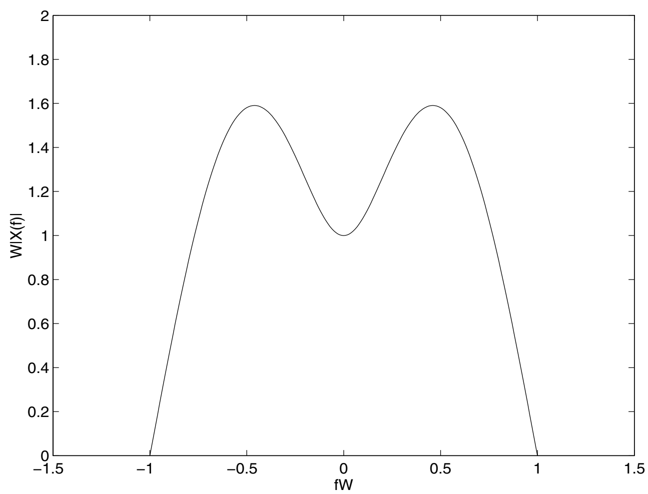
(a)
(i) $x_0 = 2,\ x_1 = 1,\ x_2 = -1,$ otherwise $x_n = 0$. Then :

$$x(t) = 2\frac{\sin(2\pi Wt)}{2\pi Wt} + \frac{\sin(2\pi W(t-1/2W))}{2\pi W(t-1/2W)} - \frac{\sin(2\pi W(t-1/W))}{2\pi W(t-1/W)}$$

and :

$$X(f) = \tfrac{1}{2W}\left[2 + e^{-j\pi f/W} - e^{-j2\pi f/W}\right], \qquad |f| \le W \Rightarrow$$
$$|X(f)| = \tfrac{1}{2W}\left[6 + 2\cos\tfrac{\pi f}{W} - 4\cos\tfrac{2\pi f}{W}\right]^{1/2}, \qquad |f| \le W$$

The plot of $|X(f)|$ is given in the following figure :



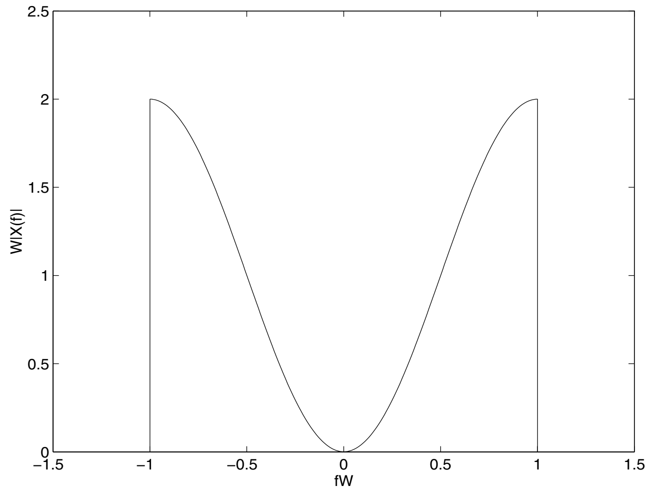
(ii) $x_{-1} = -1,\ x_0 = 2, x_1 = -1,$ otherwise $x_n = 0$. Then :

$$x(t) = 2\frac{\sin(2\pi Wt)}{2\pi Wt} - \frac{\sin(2\pi W(t+1/2W))}{2\pi W(t+1/2W)} - \frac{\sin(2\pi W(t-1/2W))}{2\pi W(t-1/2W)}$$

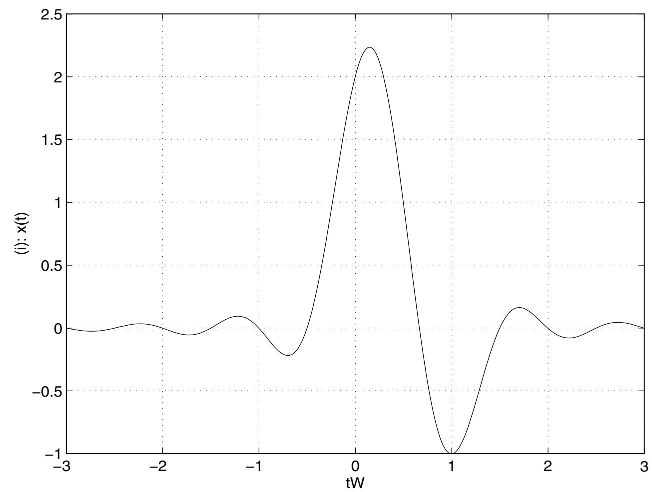
and :

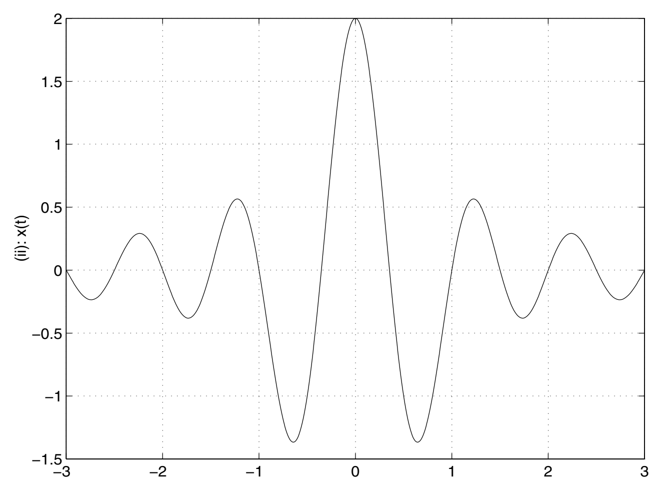
$$X(f) = \frac{1}{2W}\left[2 - e^{-j\pi f/W} - e^{+j\pi f/W}\right] = \frac{1}{2W}\left[2 - 2\cos\frac{\pi f}{W}\right] = \frac{1}{W}\left[1 - \cos\frac{\pi f}{W}\right],\ |f| \le W$$

The plot of $|X(f)|$ is given in the following figure :



(b) Based on the results obtained in part (a) :





(c) The possible received levels at the receiver are given by :
(i)

$$B_n = 2I_n + I_{n-1} - I_{n-2}$$

where $I_m = \pm 1$. Hence :

$$\begin{array}{c} P(B_n = 0) = 1/4 \\ P(B_n = -2) = 1/4 \\ P(B_n = 2) = 1/4 \\ P(B_n = -4) = 1/8 \\ P(B_n = 4) = 1/8 \end{array}$$

(ii)

$$B_n = 2I_n - I_{n-1} - I_{n+1}$$

where $I_m = \pm 1$. Hence :

$$\begin{array}{c} P(B_n = 0) = 1/4 \\ P(B_n = -2) = 1/4 \\ P(B_n = 2) = 1/4 \\ P(B_n = -4) = 1/8 \\ P(B_n = 4) = 1/8 \end{array}$$

Problem 9.11 :

The bandwidth of the bandpass channel is $W = 4$ KHz. Hence, the rate of transmission should be less or equal to 4000 symbols/sec. If a 8-QAM constellation is employed, then the required symbol rate is $R = 9600/3 = 3200$. If a signal pulse with raised cosine spectrum is used for shaping, the maximum allowable roll-off factor is determined by :

$$1600(1 + \beta) = 2000$$

which yields $\beta = 0.25$. Since β is less than 50%, we consider a larger constellation. With a 16-QAM constellation we obtain :

$$R = \frac{9600}{4} = 2400$$

and :

$$1200(1 + \beta) = 2000$$

or $\beta = 2/3$, which satisfies the required conditions. The probability of error for an M-QAM constellation is given by :

$$P_M = 1 - (1 - P_{\sqrt{M}})^2$$

where :

$$P_{\sqrt{M}} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left[\sqrt{\frac{3\mathcal{E}_{av}}{(M-1)N_0}}\right]$$

With $P_M = 10^{-6}$ we obtain $P_{\sqrt{M}} = 5 \times 10^{-7}$ and therefore using the last equation and the table of values for the $Q(\cdot)$ function, we find that the average transmitted energy is :

$$\mathcal{E}_{av} = 24.70 \times 10^{-9}$$

Note that if the desired spectral characteristic $X_{rc}(f)$ is split evenly between the transmitting and receiving filter, then the energy of the transmitting pulse is :

$$\int_{-\infty}^{\infty} g_T^2(t)dt = \int_{-\infty}^{\infty} |G_T(f)|^2 df = \int_{-\infty}^{\infty} X_{rc}(f)df = 1$$

Hence, the energy $\mathcal{E}_{av} = P_{av}T$ depends only on the amplitude of the transmitted points and the symbol interval T. Since $T = \frac{1}{2400}$, the average transmitted power is :

$$P_{av} = \frac{\mathcal{E}_{av}}{T} = 24.70 \times 10^{-9} \times 2400 = 592.8 \times 10^{-7}$$

If the points of the 16-QAM constellation are evenly spaced with minimum distance between them equal to d, then there are four points with coordinates $(\pm\frac{d}{2}, \pm\frac{d}{2})$, four points with coordinates $(\pm\frac{3d}{2}, \pm\frac{3d}{2})$, and eight points with coordinates $(\pm\frac{3d}{2}, \pm\frac{d}{2})$, or $(\pm\frac{d}{2}, \pm\frac{3d}{2})$. Thus, the average transmitted power is :

$$P_{av} = \frac{1}{2 \times 16} \sum_{i=1}^{16} (A_{mc}^2 + A_{ms}^2) = \frac{1}{32} \left[4 \times \frac{d^2}{2} + 4 \times \frac{9d^2}{2} + 8 \times \frac{10d^2}{4} \right] = \frac{5}{4} d^2$$

Since $P_{av} = 592.8 \times 10^{-7}$, we obtain

$$d = \sqrt{4 \frac{P_{av}}{5}} = 0.0069$$

Problem 9.12 :

The channel (bandpass) bandwidth is $W = 4000$ Hz. Hence, the lowpass equivalent bandwidth will extend from -2 to 2 KHz.

(a) Binary PAM with a pulse shape that has $\beta = \frac{1}{2}$. Hence :

$$\frac{1}{2T}(1 + \beta) = 2000$$

so $\frac{1}{T} = 2667$, and since $k = 1$ bit/symbols is transmitted, the bit rate is 2667 bps.

(b) Four-phase PSK with a pulse shape that has $\beta = \frac{1}{2}$. From (a) the symbol rate is $\frac{1}{T} = 2667$ and the bit rate is 5334 bps.

(c) $M = 8$ QAM with a pulse shape that has $\beta = \frac{1}{2}$. From (a), the symbol rate is $\frac{1}{T} = 2667$ and hence the bit rate $\frac{3}{T} = 8001$ bps.

(d) Binary FSK with noncoherent detection. Assuming that the frequency separation between the two frequencies is $\Delta f = \frac{1}{T}$, where $\frac{1}{T}$ is the bit rate, the two frequencies are $f_c + \frac{1}{2T}$ and $f_c - \frac{1}{2T}$. Since $W = 4000$ Hz, we may select $\frac{1}{2T} = 1000$, or, equivalently, $\frac{1}{T} = 2000$. Hence, the bit rate is 2000 bps, and the two FSK signals are orthogonal.
(e) Four FSK with noncoherent detection. In this case we need four frequencies with separation of $\frac{1}{T}$ between adjacent frequencies. We select $f_1 = f_c - \frac{1.5}{T}$, $f_2 = f_c - \frac{1}{2T}$, $f_3 = f_c + \frac{1}{2T}$, and $f_4 = f_c + \frac{1.5}{T}$, where $\frac{1}{2T} = 500$ Hz. Hence, the symbol rate is $\frac{1}{T} = 1000$ symbols per second and since each symbol carries two bits of information, the bit rate is 2000 bps.
(f) $M = 8$ FSK with noncoherent detection. In this case we require eight frequencies with frequency separation of $\frac{1}{T} = 500$ Hz for orthogonality. Since each symbol carries 3 bits of information, the bit rate is 1500 bps.

Problem 9.13 :

(a) The bandwidth of the bandpass channel is :

$$W = 3000 - 600 = 2400 \text{ Hz}$$

Since each symbol of the QPSK constellation conveys 2 bits of information, the symbol rate of transmission is :

$$R = \frac{1}{T} = \frac{2400}{2} = 1200 \text{ symbols/sec}$$

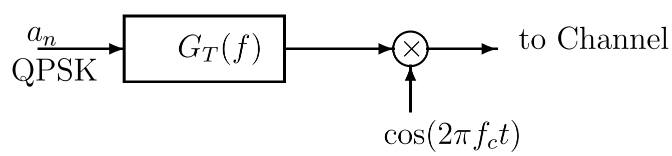
Thus, for spectral shaping we can use a signal pulse with a raised cosine spectrum and roll-off factor $\beta = 1$, since the spectral requirements will be $\frac{1}{2T}(1+\beta) = \frac{1}{T} = 1200$Hz. Hence :

$$X_{rc}(f) = \frac{T}{2}[1 + \cos(\pi T|f|)] = \frac{1}{1200}\cos^2\left(\frac{\pi|f|}{2400}\right)$$

If the desired spectral characteristic is split evenly between the transmitting filter $G_T(f)$ and the receiving filter $G_R(f)$, then

$$G_T(f) = G_R(f) = \sqrt{\frac{1}{1200}}\cos\left(\frac{\pi|f|}{2400}\right), \qquad |f| < \frac{1}{T} = 1200$$

A block diagram of the transmitter is shown in the next figure.



(b) If the bit rate is 4800 bps, then the symbol rate is

$$R = \frac{4800}{2} = 2400 \text{ symbols/sec}$$

In order to satisfy the Nyquist criterion, the the signal pulse used for spectral shaping, should have roll-off factor $\beta = 0$ with corresponding spectrum :

$$X(f) = T, \quad |f| < 1200$$

Thus, the frequency response of the transmitting filter is $G_T(f) = \sqrt{T}, \quad |f| < 1200$.