Problem 9.14 :

The bandwidth of the bandpass channel is :

$$W = 3300 - 300 = 3000 \text{ Hz}$$

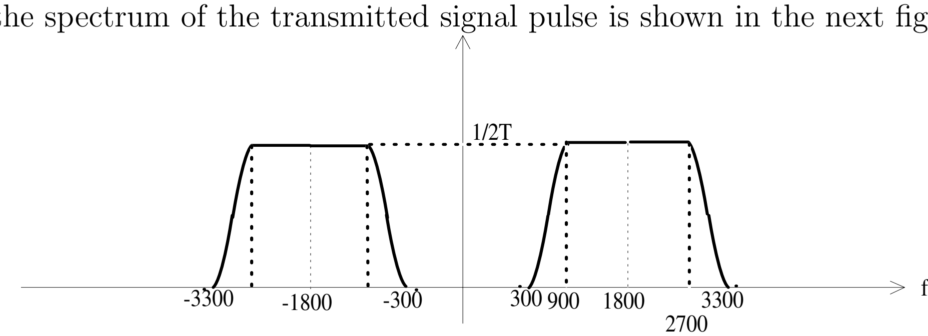
In order to transmit 9600 bps with a symbor rate $R = \frac{1}{T} = 2400$ symbols per second, the number of information bits per symbol should be :

$$k = \frac{9600}{2400} = 4$$

Hence, a $2^4 = 16$ QAM signal constellation is needed. The carrier frequency f_c is set to 1800 Hz, which is the mid-frequency of the frequency band that the bandpass channel occupies. If a pulse with raised cosine spectrum and roll-off factor β is used for spectral shaping, then for the bandpass signal with bandwidth W :

$$\frac{1}{2T}(1+\beta) = \frac{W}{2} = 1500 \Rightarrow \beta = 0.25$$

A sketch of the spectrum of the transmitted signal pulse is shown in the next figure.



Problem 9.16 :

The pulse $x(t)$ having the raised cosine spectrum given by (9-2-26/27) is :

$$x(t) = \text{sinc}(t/T)\frac{\cos(\pi\beta t/T)}{1-4\beta^2t^2/T^2}$$

The function $\text{sinc}(t/T)$ is 1 when $t=0$ and 0 when $t=nT$. Therefore, the Nyquist criterion will be satisfied as long as the function $g(t)$ is :

$$g(t) = \frac{\cos(\pi\beta t/T)}{1-4\beta^2t^2/T^2} = \begin{cases} 1 & t=0 \\ \text{bounded} & t\neq 0 \end{cases}$$

The function $g(t)$ needs to be checked only for those values of t such that $4\beta^2t^2/T^2 = 1$ or $\beta t = \frac{T}{2}$. However :

$$\lim_{\beta t\to\frac{T}{2}} \frac{\cos(\pi\beta t/T)}{1-4\beta^2t^2/T^2} = \lim_{x\to 1}\frac{\cos(\frac{\pi}{2}x)}{1-x}$$

and by using L'Hospital's rule :

$$\lim_{x\to 1}\frac{\cos(\frac{\pi}{2}x)}{1-x} = \lim_{x\to 1}\frac{\pi}{2}\sin(\frac{\pi}{2}x) = \frac{\pi}{2} < \infty$$

Hence :

$$x(nT) = \begin{cases} 1 & n=0 \\ 0 & n\neq 0 \end{cases}$$

meaning that the pulse $x(t)$ satisfies the Nyquist criterion.

Problem 9.20 :

Since the one-sided bandwidth of the ideal lowpass channel is $W = 2400$ Hz, the rate of transmission is :

$$R = 2 \times 2400 = 4800 \text{ symbols/sec}$$

(remember that PAM can be transmitted single-sideband; hence, if the lowpass channel has bandwidth from -W to W, the passband channel will have bandwidth equal to W; on the other hand, a PSK or QAM system will have passband bandwidth equal to $2W$). The number of bits per symbol is

$$k = \frac{14400}{4800} = 3$$

Hence, the number of transmitted symbols is $2^3 = 8$. If a duobinary pulse is used for transmission, then the number of possible transmitted symbols is $2M - 1 = 15$. These symbols have the form

$$b_n = 0, \pm 2d, \pm 4d, \ldots, \pm 12d$$

where $2d$ is the minimum distance between the points of the 8-PAM constellation. The probability mass function of the received symbols is

$$P(b = 2md) = \frac{8 - |m|}{64}, \qquad m = 0, \pm 1, \ldots, \pm 7$$

An upper bound of the probability of error is given by (see (9-3-18))

$$P_M < 2\left(1 - \frac{1}{M^2}\right) Q\left[\sqrt{\left(\frac{\pi}{4}\right)^2 \frac{6}{M^2 - 1} \frac{k\mathcal{E}_{b,av}}{N_0}}\right]$$

With $P_M = 10^{-6}$ and $M = 8$ we obtain

$$\frac{k\mathcal{E}_{b,av}}{N_0} = 1.3193 \times 10^3 \Longrightarrow \mathcal{E}_{b,av} = 0.088$$

Problem 9.21 :

(a) The spectrum of the baseband signal is (see (4-4-12))

$$\Phi_V(f) = \frac{1}{T}\Phi_i i(f)|X_{rc}(f)|^2 = \frac{1}{T}|X_{rc}(f)|^2$$

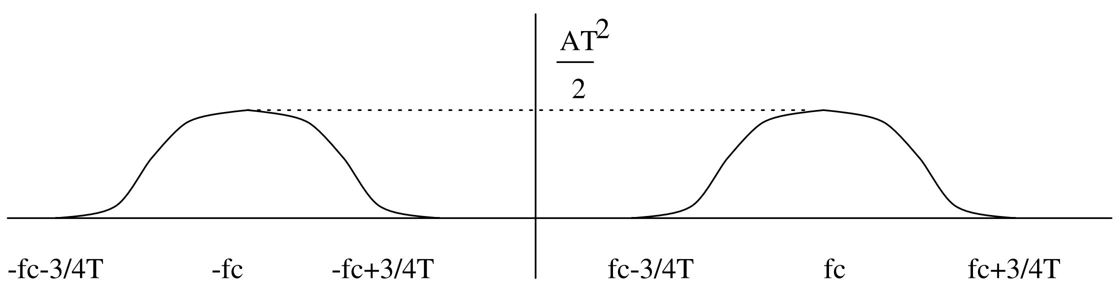
where $T = \frac{1}{2400}$ and

$$X_{rc}(f) = \begin{cases} T & 0 \leq |f| \leq \frac{1}{4T} \\ \frac{T}{2}(1+\cos(2\pi T(|f| - \frac{1}{4T})) & \frac{1}{4T} \leq |f| \leq \frac{3}{4T} \\ 0 & \text{otherwise} \end{cases}$$

If the carrier signal has the form $c(t) = A\cos(2\pi f_c t)$, then the spectrum of the DSB-SC modulated signal, $\Phi_U(f)$, is

$$\Phi_U(f) = \frac{A}{2}[\Phi_V(f - f_c) + \Phi_V(f + f_c)]$$

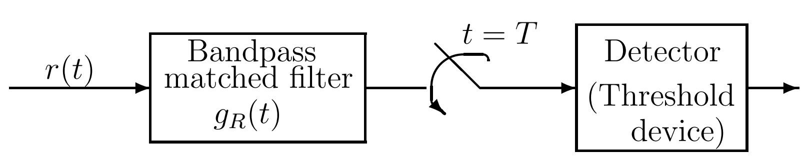
A sketch of $\Phi_U(f)$ is shown in the next figure.



(b) Assuming bandpass coherent demodulation using a matched filter, the received signal $r(t)$ is first passed through a linear filter with impulse response

$$g_R(t) = Ax_{rc}(T-t)\cos(2\pi f_c(T-t))$$

The output of the matched filter is sampled at $t = T$ and the samples are passed to the detector. The detector is a simple threshold device that decides if a binary 1 or 0 was transmitted depending on the sign of the input samples. The following figure shows a block diagram of the optimum bandpass coherent demodulator.



Problem 9.31 :

In the case where the channel distortion is fully precompensated at the transmitter, the loss of SNR is given by

$$10\log L_1, \quad \text{with} \quad L_1 = \int_{-W}^{W} \frac{X_{rc}(f)}{|C(f)|^2}$$

whereas in the case of the equally split filters, the loss of SNR is given by

$$10\log [L_2]^2, \quad \text{with} \quad L_2 = \int_{-W}^{W} \frac{X_{rc}(f)}{|C(f)|}$$

Assuming that $1/T = W$, so that we have a raised cosine characteristic with $\beta = 0$, we have

$$X_{rc}(f) = \frac{1}{2W}\left[1 + \cos\frac{\pi|f|}{W}\right]$$

Then

$$\begin{aligned} L_1 &= 2\int_0^W \tfrac{1}{2W}\tfrac{[1+\cos\frac{\pi f}{W}]}{|C(f)|^2} \\ &= 2\left[\int_0^{W/2} \tfrac{1}{2W}\tfrac{[1+\cos\frac{\pi f}{W}]}{1} + \int_{W/2}^{W} \tfrac{1}{2W}\tfrac{[1+\cos\frac{\pi f}{W}]}{1/4}\right] \\ &= \tfrac{5\pi-6}{2\pi} \end{aligned}$$

Hence, the loss for the first type of filters is $10\log L_1 = 1.89$ dB.
In a similar way,

$$\begin{aligned} L_2 &= 2\int_0^W \tfrac{1}{2W}\tfrac{[1+\cos\frac{\pi f}{W}]}{|C(f)|} \\ &= 2\left[\int_0^{W/2} \tfrac{1}{2W}\tfrac{[1+\cos\frac{\pi f}{W}]}{1} + \int_{W/2}^{W} \tfrac{1}{2W}\tfrac{[1+\cos\frac{\pi f}{W}]}{1/2}\right] \\ &= \tfrac{3\pi-2}{2\pi} \end{aligned}$$

Hence, the loss for the second type of filters is $10\log[L_2]^2 = 1.45$ dB. As expected, the second type of filters which split the channel characteristics between the transmitter and the receiver exhibit a smaller SNR loss.