Problem 10.2 :

(a) If the transmitted signal is :

$$r(t) = \sum_{n=-\infty}^{\infty} I_n h(t - nT) + n(t)$$

then the output of the receiving filter is :

$$y(t) = \sum_{n=-\infty}^{\infty} I_n x(t - nT) + \nu(t)$$

where $x(t) = h(t) \star h(t)$ and $\nu(t) = n(t) \star h(t)$. If the sampling time is off by 10%, then the samples at the output of the correlator are taken at $t = (m \pm \frac{1}{10})T$. Assuming that $t = (m - \frac{1}{10})T$ without loss of generality, then the sampled sequence is :

$$y_m = \sum_{n=-\infty}^{\infty} I_n x((m - \frac{1}{10}T - nT) + \nu((m - \frac{1}{10})T)$$

If the signal pulse is rectangular with amplitude A and duration T, then $\sum_{n=-\infty}^{\infty} I_n x((m - \frac{1}{10}T - nT)$ is nonzero only for $n = m$ and $n = m - 1$ and therefore, the sampled sequence is given by :

$$\begin{aligned} y_m &= I_m x(-\frac{1}{10}T) + I_{m-1} x(T - \frac{1}{10}T) + \nu((m - \frac{1}{10})T) \\ &= \frac{9}{10} I_m A^2 T + I_{m-1} \frac{1}{10} A^2 T + \nu((m - \frac{1}{10})T) \end{aligned}$$

The variance of the noise is :

$$\sigma_\nu^2 = \frac{N_0}{2} A^2 T$$

and therefore, the SNR is :

$$\text{SNR} = \left(\frac{9}{10}\right)^2 \frac{2(A^2T)^2}{N_0 A^2 T} = \frac{81}{100} \frac{2A^2T}{N_0}$$

As it is observed, there is a loss of $10 \log_{10} \frac{81}{100} = -0.9151$ dB due to the mistiming.

(b) Recall from part (a) that the sampled sequence is

$$y_m = \frac{9}{10} I_m A^2 T + I_{m-1} \frac{1}{10} A^2 T + \nu_m$$

The term $I_{m-1} \frac{A^2T}{10}$ expresses the ISI introduced to the system. If $I_m = 1$ is transmitted, then the probability of error is

$$\begin{aligned} P(e|I_m = 1) &= \frac{1}{2} P(e|I_m = 1, I_{m-1} = 1) + \frac{1}{2} P(e|I_m = 1, I_{m-1} = -1) \\ &= \frac{1}{2\sqrt{\pi N_0 A^2 T}} \int_{-\infty}^{-A^2T} e^{-\frac{\nu^2}{N_0 A^2 T}} d\nu + \frac{1}{2\sqrt{\pi N_0 A^2 T}} \int_{-\infty}^{-\frac{8}{10}A^2T} e^{-\frac{\nu^2}{N_0 A^2 T}} d\nu \\ &= \frac{1}{2} Q\left[\sqrt{\frac{2A^2T}{N_0}}\right] + \frac{1}{2} Q\left[\sqrt{\left(\frac{8}{10}\right)^2 \frac{2A^2T}{N_0}}\right] \end{aligned}$$

Since the symbols of the binary PAM system are equiprobable the previous derived expression is the probability of error when a symbol by symbol detector is employed. Comparing this with the probability of error of a system with no ISI, we observe that there is an increase of the probability of error by

$$P_{\text{diff}}(e) = \frac{1}{2} Q\left[\sqrt{\left(\frac{8}{10}\right)^2 \frac{2A^2T}{N_0}}\right] - \frac{1}{2} Q\left[\sqrt{\frac{2A^2T}{N_0}}\right]$$

Problem 10.3 :

(a) Taking the inverse Fourier transform of $H(f)$, we obtain :

$$h(t) = \mathcal{F}^{-1}[H(f)] = \delta(t) + \frac{\alpha}{2} \delta(t - t_0) + \frac{\alpha}{2} \delta(t + t_0)$$

Hence,

$$y(t) = s(t) \star h(t) = s(t) + \frac{\alpha}{2} s(t - t_0) + \frac{\alpha}{2} s(t + t_0)$$

(b) If the signal $s(t)$ is used to modulate the sequence $\{I_n\}$, then the transmitted signal is :

$$u(t) = \sum_{n=-\infty}^{\infty} I_n s(t - nT)$$

The received signal is the convolution of $u(t)$ with $h(t)$. Hence,

$$\begin{aligned} y(t) &= u(t) \star h(t) = \left(\sum_{n=-\infty}^{\infty} I_n s(t-nT) \right) \star \left(\delta(t) + \frac{\alpha}{2}\delta(t-t_0) + \frac{\alpha}{2}\delta(t+t_0) \right) \\ &= \sum_{n=-\infty}^{\infty} I_n s(t-nT) + \frac{\alpha}{2} \sum_{n=-\infty}^{\infty} I_n s(t-t_0-nT) + \frac{\alpha}{2} \sum_{n=-\infty}^{\infty} I_n s(t+t_0-nT) \end{aligned}$$

Thus, the output of the matched filter $s(-t)$ at the time instant t_1 is :

$$\begin{aligned} w(t_1) &= \sum_{n=-\infty}^{\infty} I_n \int_{-\infty}^{\infty} s(\tau - nT) s(\tau - t_1) d\tau \\ &\quad + \frac{\alpha}{2} \sum_{n=-\infty}^{\infty} I_n \int_{-\infty}^{\infty} s(\tau - t_0 - nT) s(\tau - t_1) d\tau \\ &\quad + \frac{\alpha}{2} \sum_{n=-\infty}^{\infty} I_n \int_{-\infty}^{\infty} s(\tau + t_0 - nT) s(\tau - t_1) d\tau \end{aligned}$$

If we denote the signal $s(t) \star s(t)$ by $x(t)$, then the output of the matched filter at $t_1 = kT$ is :

$$\begin{aligned} w(kT) &= \sum_{n=-\infty}^{\infty} I_n x(kT - nT) \\ &\quad + \frac{\alpha}{2} \sum_{n=-\infty}^{\infty} I_n x(kT - t_0 - nT) + \frac{\alpha}{2} \sum_{n=-\infty}^{\infty} I_n x(kT + t_0 - nT) \end{aligned}$$

(c) With $t_0 = T$ and $k = n$ in the previous equation, we obtain :

$$\begin{aligned} w_k &= I_k x_0 + \sum_{n \neq k} I_n x_{k-n} \\ &\quad + \frac{\alpha}{2} I_k x_{-1} + \frac{\alpha}{2} \sum_{n \neq k} I_n x_{k-n-1} + \frac{\alpha}{2} I_k x_1 + \frac{\alpha}{2} \sum_{n \neq k} I_n x_{k-n+1} \\ &= I_k \left(x_0 + \frac{\alpha}{2} x_{-1} + \frac{\alpha}{2} x_1 \right) + \sum_{n \neq k} I_n \left[x_{k-n} + \frac{\alpha}{2} x_{k-n-1} + \frac{\alpha}{2} x_{k-n+1} \right] \end{aligned}$$

The terms under the summation is the ISI introduced by the channel. If the signal $s(t)$ is designed so as to satisfy the Nyquist criterion, then :

$$x_k = 0, \ k \neq 0$$

and the above expression simplifies to :

$$w_k = I_k + \frac{\alpha}{2}(I_{k+1} + I_{k-1})$$

Problem 10.8 :

(a) The output of the matched filter at the time instant mT is :

$$y_m = \sum_k I_m x_{k-m} + \nu_m = I_m + \frac{1}{4} I_{m-1} + \nu_m$$

The autocorrelation function of the noise samples ν_m is

$$E[\nu_k \nu_j] = \frac{N_0}{2} x_{k-j}$$

Thus, the variance of the noise is

$$\sigma_\nu^2 = \frac{N_0}{2} x_0 = \frac{N_0}{2}$$

If a symbol by symbol detector is employed and we assume that the symbols $I_m = I_{m-1} = \sqrt{\mathcal{E}_b}$ have been transmitted, then the probability of error $P(e|I_m = I_{m-1} = \sqrt{\mathcal{E}_b})$ is :

$$P(e|I_m = I_{m-1} = \sqrt{\mathcal{E}_b}) \quad = \quad P(y_m < 0 | I_m = I_{m-1} = \sqrt{\mathcal{E}_b})$$

$$= \quad P(\nu_m < -\frac{5}{4}\sqrt{\mathcal{E}_b}) = \frac{1}{\sqrt{\pi N_0}}\int_{-\infty}^{-\frac{5}{4}\sqrt{\mathcal{E}_b}} e^{-\frac{\nu_m^2}{N_0}} d\nu_m$$

$$= \quad \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-\frac{5}{4}\sqrt{\frac{2\mathcal{E}_b}{N_0}}} e^{-\frac{\nu^2}{2}} d\nu = Q\left[\frac{5}{4}\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right]$$

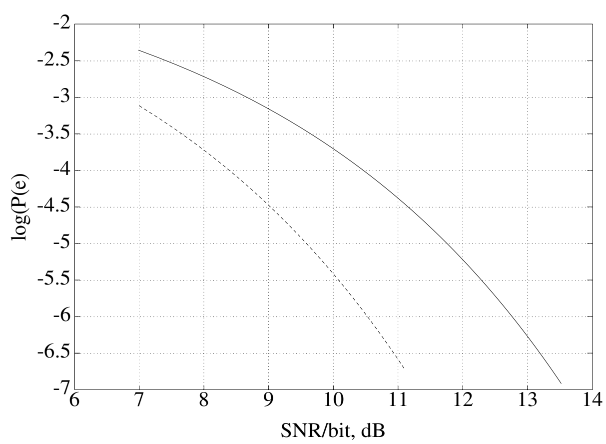
If however $I_{m-1} = -\sqrt{\mathcal{E}_b}$, then :

$$P(e|I_m = \sqrt{\mathcal{E}_b}, I_{m-1} = -\sqrt{\mathcal{E}_b}) = P(\frac{3}{4}\sqrt{\mathcal{E}_b} + \nu_m < 0) = Q\left[\frac{3}{4}\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right]$$

Since the two symbols $\sqrt{\mathcal{E}_b}$, $-\sqrt{\mathcal{E}_b}$ are used with equal probability, we conclude that :

$$\begin{aligned} P(e) &= P(e|I_m = \sqrt{\mathcal{E}_b}) = P(e|I_m = -\sqrt{\mathcal{E}_b}) \\ &= \frac{1}{2}Q\left[\frac{5}{4}\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right] + \frac{1}{2}Q\left[\frac{3}{4}\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right] \end{aligned}$$

(b) In the next figure we plot the error probability obtained in part (a) ($\log_{10}(P(e))$) vs. the SNR per bit and the error probability for the case of no ISI. As it observed from the figure, the relative difference in SNR of the error probability of 10^{-6} is 2 dB.



Problem 10.13 :

The optimum tap coefficients of the zero-force equalizer can be found by solving the system:

$$\begin{pmatrix} 1.0 & 0.3 & 0.0 \\ 0.2 & 1.0 & 0.3 \\ 0.0 & 0.2 & 1.0 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Hence,

$$\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} -0.3409 \\ 1.1364 \\ -0.2273 \end{pmatrix}$$

The output of the equalizer is :

$$q_m = \begin{cases} 0 & m \leq -3 \\ c_{-1}x_{-1} = -0.1023 & m = -2 \\ 0 & m = -1 \\ 1 & m = 0 \\ 0 & m = 1 \\ c_1 x_1 = -0.0455 & m = 2 \\ 0 & m \geq 3 \end{cases}$$

Hence, the residual ISI sequence is :

$$\text{residual ISI} = \{\ldots, 0, -0.1023, 0, 0, 0, -0.0455, 0, \ldots\}$$