Problem 4.11 :

(a) As an orthonormal set of basis functions we consider the set

$$f_1(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & \text{o.w} \end{cases} \qquad f_2(t) = \begin{cases} 1 & 1 \le t < 2 \\ 0 & \text{o.w} \end{cases}$$

$$f_3(t) = \begin{cases} 1 & 2 \le t < 3 \\ 0 & \text{o.w} \end{cases} \qquad f_4(t) = \begin{cases} 1 & 3 \le t < 4 \\ 0 & \text{o.w} \end{cases}$$

In matrix notation, the four waveforms can be represented as

$$\begin{pmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \\ s_4(t) \end{pmatrix} = \begin{pmatrix} 2 & -1 & -1 & -1 \\ -2 & 1 & 1 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & -2 & -2 & 2 \end{pmatrix} \begin{pmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \\ f_4(t) \end{pmatrix}$$

Note that the rank of the transformation matrix is 4 and therefore, the dimensionality of the waveforms is 4

(b) The representation vectors are

$$\begin{aligned} \mathbf{s}_1 &= \begin{bmatrix} 2 & -1 & -1 & -1 \end{bmatrix} \\ \mathbf{s}_2 &= \begin{bmatrix} -2 & 1 & 1 & 0 \end{bmatrix} \\ \mathbf{s}_3 &= \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix} \\ \mathbf{s}_4 &= \begin{bmatrix} 1 & -2 & -2 & 2 \end{bmatrix} \end{aligned}$$

(c) The distance between the first and the second vector is:

$$d_{1,2} = \sqrt{|\mathbf{s}_1 - \mathbf{s}_2|^2} = \sqrt{\left|\begin{bmatrix} 4 & -2 & -2 & -1 \end{bmatrix}\right|^2} = \sqrt{25}$$

Similarly we find that :

$$\begin{aligned} d_{1,3} &= \sqrt{|\mathbf{s}_1 - \mathbf{s}_3|^2} = \sqrt{\left|\begin{bmatrix} 1 & 0 & -2 & 0 \end{bmatrix}\right|^2} = \sqrt{5} \\ d_{1,4} &= \sqrt{|\mathbf{s}_1 - \mathbf{s}_4|^2} = \sqrt{\left|\begin{bmatrix} 1 & 1 & 1 & -3 \end{bmatrix}\right|^2} = \sqrt{12} \\ d_{2,3} &= \sqrt{|\mathbf{s}_2 - \mathbf{s}_3|^2} = \sqrt{\left|\begin{bmatrix} -3 & 2 & 0 & 1 \end{bmatrix}\right|^2} = \sqrt{14} \\ d_{2,4} &= \sqrt{|\mathbf{s}_2 - \mathbf{s}_4|^2} = \sqrt{\left|\begin{bmatrix} -3 & 3 & 3 & -2 \end{bmatrix}\right|^2} = \sqrt{31} \\ d_{3,4} &= \sqrt{|\mathbf{s}_3 - \mathbf{s}_4|^2} = \sqrt{\left|\begin{bmatrix} 0 & 1 & 3 & -3 \end{bmatrix}\right|^2} = \sqrt{19} \end{aligned}$$

Thus, the minimum distance between any pair of vectors is $d_{\min} = \sqrt{5}$.

Problem 4.13 :

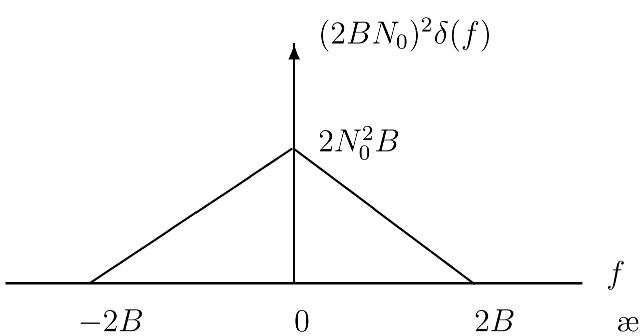
The power spectral density of $X(t)$ corresponds to : $\phi_{xx}(t) = 2BN_0 \frac{\sin 2\pi Bt}{2\pi Bt}$. From the result of Problem 2.14 :

$$\phi_{yy}(\tau) = \phi_{xx}^2(0) + 2\phi_{xx}^2(\tau) = (2BN_0)^2 + 8B^2N_0^2 \left(\frac{\sin 2\pi Bt}{2\pi Bt}\right)^2$$

Also :

$$\Phi_{yy}(f) = \phi_{xx}^2(0)\delta(f) + 2\Phi_{xx}(f) * \Phi_{xx}(f)$$

The following figure shows the power spectral density of $Y(t)$:



Problem 4.23 :

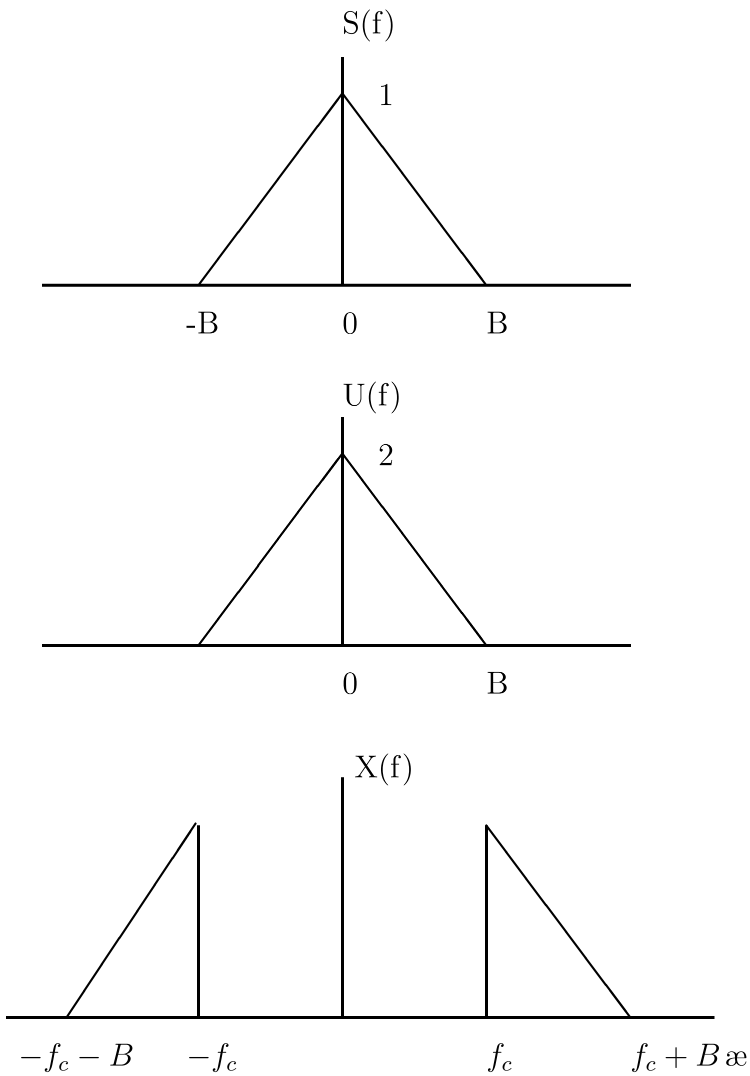
$x(t) = Re\left[u(t)\exp\left(j2\pi f_c t\right)\right]$ where $u(t) = s(t) \pm j\hat{s}(t)$. Hence :

$$U(f) = S(f) \pm j\hat{S}(f) \qquad \text{where } \hat{S}(f) = \left\{ \begin{array}{cc} -jS(f), & f > 0 \\ jS(f), & f < 0 \end{array} \right\}$$

So :

$$U(f) = \left\{ \begin{array}{cc} S(f) \pm S(f), & f > 0 \\ S(f) \mp S(f), & f < 0 \end{array} \right\} = \left\{ \begin{array}{cc} 2S(f) \text{ or } 0, & f > 0 \\ 0 \text{ or } 2S(f), & f < 0 \end{array} \right\}$$

Since the lowpass equivalent of $x(t)$ is single-sideband, we conclude that $x(t)$ is a single-sideband signal, too. Suppose, for example, that $s(t)$ has the following spectrum. Then, the spectra of the signals $u(t)$ (shown in the figure for the case $u(t) = s(t) + j\hat{s}(t)$) and $x(t)$ are single-sideband


S(f)
1
-B
0
B
U(f)
2
0
B
X(f)
$-f_c - B$
$-f_c$
f_c
$f_c + B$ æ

Problem 4.30 :

The 16-QAM signal is represented as $s(t) = I_n \cos 2\pi f t + Q_n \sin 2\pi f t$, where $I_n = \{\pm 1, \pm 3\}$, $Q_n = \{\pm 1, \pm 3\}$. A superposition of two 4-QAM (4-PSK) signals is :

$$s(t) = G\left[A_n \cos 2\pi f t + B_n \sin 2\pi f t\right] + C_n \cos 2\pi f t + C_n \sin 2\pi f t$$

where $A_n, B_{n,} C_n, D_n = \{\pm 1\}$. Clearly : $I_n = GA_n + C_n,\ Q_n = GB_n + D_n$. From these equations it is easy to see that $G = 2$ gives the requires equivalence.