Problem 6.13 :

Assume that the signal $u_m(t)$ is the input to the Costas loop. Then $u_m(t)$ is multiplied by $\cos(2\pi f_c t + \hat{\phi})$ and $\sin(2\pi f_c t + \hat{\phi})$, where $\cos(2\pi f_c t + \hat{\phi})$ is the output of the VCO. Hence :

$$\begin{aligned}
& u_{mc}(t) \\
&= A_m g_T(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \hat{\phi}) - A_m \hat{g}_T(t)\sin(2\pi f_c t)\cos(2\pi f_c t + \hat{\phi}) \\
&= \frac{A_m g_T(t)}{2}\left[\cos(2\pi 2 f_c t + \hat{\phi}) + \cos(\hat{\phi})\right] - \frac{A_m \hat{g}_T(t)}{2}\left[\sin(2\pi 2 f_c t + \hat{\phi}) - \sin(\hat{\phi})\right] \\
& u_{ms}(t) \\
&= A_m g_T(t)\cos(2\pi f_c t)\sin(2\pi f_c t + \hat{\phi}) - A_m \hat{g}_T(t)\sin(2\pi f_c t)\sin(2\pi f_c t + \hat{\phi}) \\
&= \frac{A_m g_T(t)}{2}\left[\sin(2\pi 2 f_c t + \hat{\phi}) + \sin(\hat{\phi})\right] - \frac{A_m \hat{g}_T(t)}{2}\left[\cos(\hat{\phi}) - \cos(2\pi 2 f_c t + \hat{\phi})\right]
\end{aligned}$$

The lowpass filters of the Costas loop will reject the double frequency components, so that :

$$\begin{aligned}
y_{mc}(t) &= \frac{A_m g_T(t)}{2}\cos(\hat{\phi}) + \frac{A_m \hat{g}_T(t)}{2}\sin(\hat{\phi}) \\
y_{ms}(t) &= \frac{A_m g_T(t)}{2}\sin(\hat{\phi}) - \frac{A_m \hat{g}_T(t)}{2}\cos(\hat{\phi})
\end{aligned}$$

Note that when the carrier phase has been extracted correctly, $\hat{\phi} = 0$ and therefore :

$$y_{mc}(t) = \frac{A_m g_T(t)}{2}, \qquad y_{ms}(t) = -\frac{A_m \hat{g}_T(t)}{2}$$

If the second signal, $y_{ms}(t)$ is passed through a Hilbert transformer, then :

$$\hat{y}_{ms}(t) = -\frac{A_m \hat{\hat{g}}_T(t)}{2} = \frac{A_m g_T(t)}{2}$$

and by adding this signal to $y_{mc}(t)$ we obtain the original unmodulated signal.

Problem 6.16 :

The PDF of the carrier phase error ϕ_e, is given by :

$$p(\phi_e) = \frac{1}{\sqrt{2\pi}\sigma_\phi} e^{-\frac{\phi_e^2}{2\sigma_\phi^2}}$$

Thus the average probability of error is :

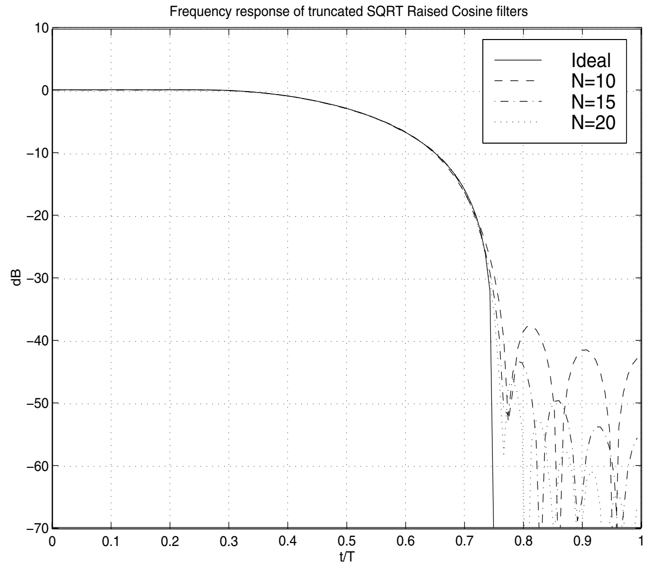
$$\begin{aligned}
\bar{P}_2 &= \int_{-\infty}^{\infty} P_2(\phi_e) p(\phi_e) d\phi_e \\
&= \int_{-\infty}^{\infty} Q\left[\sqrt{\frac{2\mathcal{E}_b}{N_0}\cos^2\phi_e}\right] p(\phi_e) d\phi_e \\
&= \frac{1}{2\pi\sigma_\phi} \int_{-\infty}^{\infty} \int_{\sqrt{\frac{2\mathcal{E}_b}{N_0}\cos^2\phi_e}}^{\infty} \exp\left[-\frac{1}{2}\left(x^2 + \frac{\phi_e^2}{\sigma_\phi^2}\right)\right] dx d\phi_e
\end{aligned}$$

Problem 9.6 :

(a)(b) In order to calculate the frequency response based on the impulse response, we need the values of the impulse response at $t = 0, \pm T/2$, which are not given directly by the expression of Problem 9.5. Using L'Hospital's rule it is straightforward to show that:

$$x(0) = \frac{1}{2} + \frac{2}{\pi}, \quad x(\pm T/2) = \frac{\sqrt{2}}{2}\frac{(2+\pi)}{2\pi}$$

Then, the frequency response of the filters with $N = 10, 15, 20$ compared to the frequency response of the ideal square-root raised cosine filter are depicted in the following figure.



As we see, there is no significant difference in the passband area of the filters, but the realizable, truncated filters do have spectral sidelobes outside their $(1+\beta)/T$ nominal bandwidth. Still, depending on how much residual ISI an application can tolerate, even the $N = 10$ filter appears an acceptable approximation of the ideal (non-realizable) square-root raised cosine filter.

Problem 9.10 :

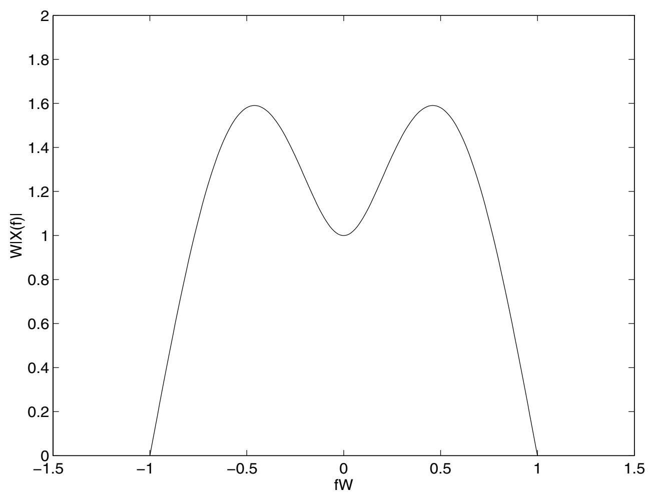
(a)
(i) $x_0 = 2,\ x_1 = 1,\ x_2 = -1,$ otherwise $x_n = 0$. Then :

$$x(t) = 2\frac{\sin(2\pi Wt)}{2\pi Wt} + \frac{\sin(2\pi W(t-1/2W))}{2\pi W(t-1/2W)} - \frac{\sin(2\pi W(t-1/W))}{2\pi W(t-1/W)}$$

and :

$$\begin{array}{rl} X(f) = \frac{1}{2W}\left[2 + e^{-j\pi f/W} - e^{-j2\pi f/W}\right], & |f| \leq W \Rightarrow \\ |X(f)| = \frac{1}{2W}\left[6 + 2\cos\frac{\pi f}{W} - 4\cos\frac{2\pi f}{W}\right]^{1/2}, & |f| \leq W \end{array}$$

The plot of $|X(f)|$ is given in the following figure :



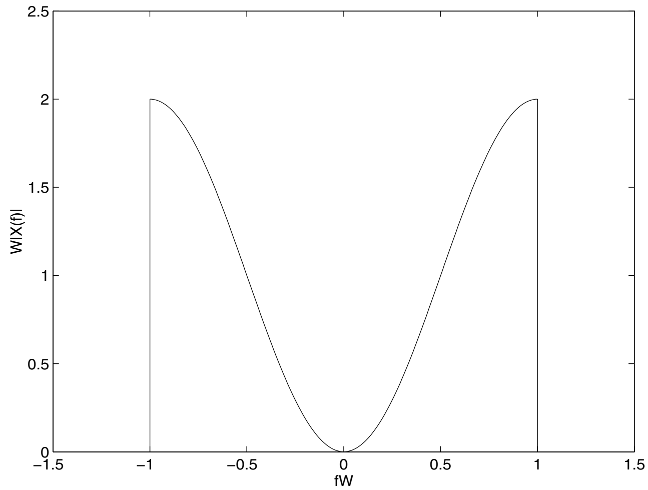
(ii) $x_{-1} = -1,\ x_0 = 2, x_1 = -1,$ otherwise $x_n = 0$. Then :

$$x(t) = 2\frac{\sin(2\pi Wt)}{2\pi Wt} - \frac{\sin(2\pi W(t+1/2W))}{2\pi W(t+1/2W)} - \frac{\sin(2\pi W(t-1/2W))}{2\pi W(t-1/2W)}$$

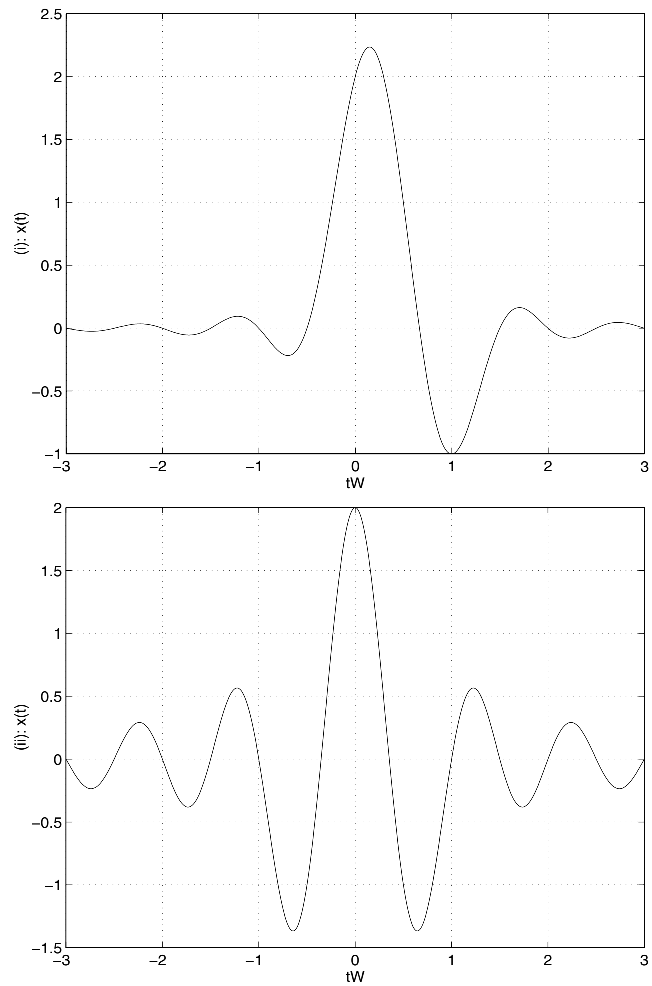
and :

$$X(f) = \frac{1}{2W}\left[2 - e^{-j\pi f/W} - e^{+j\pi f/W}\right] = \frac{1}{2W}\left[2 - 2\cos\frac{\pi f}{W}\right] = \frac{1}{W}\left[1 - \cos\frac{\pi f}{W}\right],\ |f| \leq W$$

The plot of $|X(f)|$ is given in the following figure :



(b) Based on the results obtained in part (a) :



(c) The possible received levels at the receiver are given by :
(i)

$$B_n = 2I_n + I_{n-1} - I_{n-2}$$

where $I_m = \pm 1$. Hence :

$$\begin{array}{c} P(B_n = 0) = 1/4 \\ P(B_n = -2) = 1/4 \\ P(B_n = 2) = 1/4 \\ P(B_n = -4) = 1/8 \\ P(B_n = 4) = 1/8 \end{array}$$

(ii)

$$B_n = 2I_n - I_{n-1} - I_{n+1}$$

where $I_m = \pm 1$. Hence :

$$\begin{array}{c} P(B_n = 0) = 1/4 \\ P(B_n = -2) = 1/4 \\ P(B_n = 2) = 1/4 \\ P(B_n = -4) = 1/8 \\ P(B_n = 4) = 1/8 \end{array}$$

Problem 9.11 :

The bandwidth of the bandpass channel is $W = 4$ KHz. Hence, the rate of transmission should be less or equal to 4000 symbols/sec. If a 8-QAM constellation is employed, then the required symbol rate is $R = 9600/3 = 3200$. If a signal pulse with raised cosine spectrum is used for shaping, the maximum allowable roll-off factor is determined by :

$$1600(1 + \beta) = 2000$$

which yields $\beta = 0.25$. Since β is less than 50%, we consider a larger constellation. With a 16-QAM constellation we obtain :

$$R = \frac{9600}{4} = 2400$$

and :

$$1200(1 + \beta) = 2000$$

or $\beta = 2/3$, which satisfies the required conditions. The probability of error for an M-QAM constellation is given by :

$$P_M = 1 - (1 - P_{\sqrt{M}})^2$$

where :

$$P_{\sqrt{M}} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left[\sqrt{\frac{3\mathcal{E}_{av}}{(M-1)N_0}}\right]$$

With $P_M = 10^{-6}$ we obtain $P_{\sqrt{M}} = 5 \times 10^{-7}$ and therefore using the last equation and the table of values for the $Q(\cdot)$ function, we find that the average transmitted energy is :

$$\mathcal{E}_{av} = 24.70 \times 10^{-9}$$

Note that if the desired spectral characteristic $X_{rc}(f)$ is split evenly between the transmitting and receiving filter, then the energy of the transmitting pulse is :

$$\int_{-\infty}^{\infty} g_T^2(t)dt = \int_{-\infty}^{\infty} |G_T(f)|^2 df = \int_{-\infty}^{\infty} X_{rc}(f)df = 1$$

Hence, the energy $\mathcal{E}_{av} = P_{av}T$ depends only on the amplitude of the transmitted points and the symbol interval T. Since $T = \frac{1}{2400}$, the average transmitted power is :

$$P_{av} = \frac{\mathcal{E}_{av}}{T} = 24.70 \times 10^{-9} \times 2400 = 592.8 \times 10^{-7}$$

If the points of the 16-QAM constellation are evenly spaced with minimum distance between them equal to d, then there are four points with coordinates $(\pm\frac{d}{2}, \pm\frac{d}{2})$, four points with coordinates $(\pm\frac{3d}{2}, \pm\frac{3d}{2})$, and eight points with coordinates $(\pm\frac{3d}{2}, \pm\frac{d}{2})$, or $(\pm\frac{d}{2}, \pm\frac{3d}{2})$. Thus, the average transmitted power is :

$$P_{av} = \frac{1}{2 \times 16}\sum_{i=1}^{16}(A_{mc}^2 + A_{ms}^2) = \frac{1}{32}\left[4 \times \frac{d^2}{2} + 4 \times \frac{9d^2}{2} + 8 \times \frac{10d^2}{4}\right] = \frac{5}{4}d^2$$

Since $P_{av} = 592.8 \times 10^{-7}$, we obtain

$$d = \sqrt{4\frac{P_{av}}{5}} = 0.0069$$