

Problem 9.12 :

The channel (bandpass) bandwidth is $W = 4000$ Hz. Hence, the lowpass equivalent bandwidth will extend from -2 to 2 KHz.

(a) Binary PAM with a pulse shape that has $\beta = \frac{1}{2}$. Hence :

$$\frac{1}{2T}(1 + \beta) = 2000$$

so $\frac{1}{T} = 2667$, and since $k = 1$ bit/symbols is transmitted, the bit rate is 2667 bps.

(b) Four-phase PSK with a pulse shape that has $\beta = \frac{1}{2}$. From (a) the symbol rate is $\frac{1}{T} = 2667$ and the bit rate is 5334 bps.

(c) $M = 8$ QAM with a pulse shape that has $\beta = \frac{1}{2}$. From (a), the symbol rate is $\frac{1}{T} = 2667$ and hence the bit rate $\frac{3}{T} = 8001$ bps.

(d) Binary FSK with noncoherent detection. Assuming that the frequency separation between the two frequencies is $\Delta f = \frac{1}{T}$, where $\frac{1}{T}$ is the bit rate, the two frequencies are $f_c + \frac{1}{2T}$ and $f_c - \frac{1}{2T}$. Since $W = 4000$ Hz, we may select $\frac{1}{2T} = 1000$, or, equivalently, $\frac{1}{T} = 2000$. Hence, the bit rate is 2000 bps, and the two FSK signals are orthogonal.

(e) Four FSK with noncoherent detection. In this case we need four frequencies with separation of $\frac{1}{T}$ between adjacent frequencies. We select $f_1 = f_c - \frac{1.5}{T}$, $f_2 = f_c - \frac{1}{2T}$, $f_3 = f_c + \frac{1}{2T}$, and $f_4 = f_c + \frac{1.5}{T}$, where $\frac{1}{2T} = 500$ Hz. Hence, the symbol rate is $\frac{1}{T} = 1000$ symbols per second and since each symbol carries two bits of information, the bit rate is 2000 bps.

(f) $M = 8$ FSK with noncoherent detection. In this case we require eight frequencies with frequency separation of $\frac{1}{T} = 500$ Hz for orthogonality. Since each symbol carries 3 bits of information, the bit rate is 1500 bps.

Problem 9.14 :

The bandwidth of the bandpass channel is :

$$W = 3300 - 300 = 3000 \text{ Hz}$$

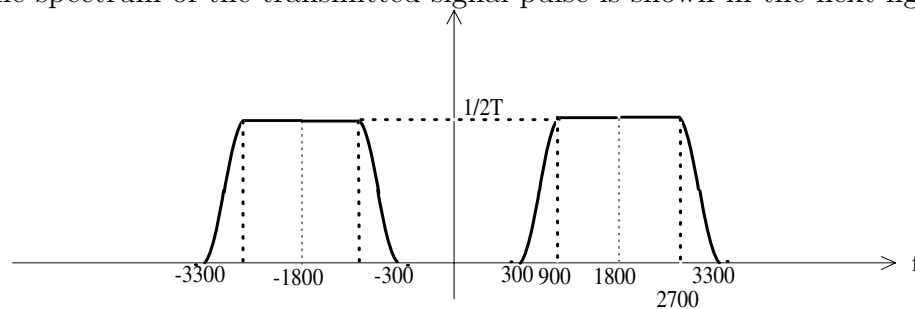
In order to transmit 9600 bps with a symbol rate $R = \frac{1}{T} = 2400$ symbols per second, the number of information bits per symbol should be :

$$k = \frac{9600}{2400} = 4$$

Hence, a $2^4 = 16$ QAM signal constellation is needed. The carrier frequency f_c is set to 1800 Hz, which is the mid-frequency of the frequency band that the bandpass channel occupies. If a pulse with raised cosine spectrum and roll-off factor β is used for spectral shaping, then for the bandpass signal with bandwidth W :

$$\frac{1}{2T}(1 + \beta) = \frac{W}{2} = 1500 \Rightarrow \beta = 0.25$$

A sketch of the spectrum of the transmitted signal pulse is shown in the next figure.



Problem 9.16 :

The pulse $x(t)$ having the raised cosine spectrum given by (9-2-26/27) is :

$$x(t) = \text{sinc}(t/T) \frac{\cos(\pi\beta t/T)}{1 - 4\beta^2 t^2/T^2}$$

The function $\text{sinc}(t/T)$ is 1 when $t = 0$ and 0 when $t = nT$. Therefore, the Nyquist criterion will be satisfied as long as the function $g(t)$ is :

$$g(t) = \frac{\cos(\pi\beta t/T)}{1 - 4\beta^2 t^2/T^2} = \begin{cases} 1 & t = 0 \\ \text{bounded} & t \neq 0 \end{cases}$$

The function $g(t)$ needs to be checked only for those values of t such that $4\beta^2 t^2/T^2 = 1$ or $\beta t = \frac{T}{2}$. However :

$$\lim_{\beta t \rightarrow \frac{T}{2}} \frac{\cos(\pi\beta t/T)}{1 - 4\beta^2 t^2/T^2} = \lim_{x \rightarrow 1} \frac{\cos(\frac{\pi}{2}x)}{1 - x}$$

and by using L'Hospital's rule :

$$\lim_{x \rightarrow 1} \frac{\cos(\frac{\pi}{2}x)}{1 - x} = \lim_{x \rightarrow 1} \frac{\pi}{2} \sin(\frac{\pi}{2}x) = \frac{\pi}{2} < \infty$$

Hence :

$$x(nT) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

meaning that the pulse $x(t)$ satisfies the Nyquist criterion.

Problem 9.20 :

Since the one-sided bandwidth of the ideal lowpass channel is $W = 2400$ Hz, the rate of transmission is :

$$R = 2 \times 2400 = 4800 \text{ symbols/sec}$$

(remember that PAM can be transmitted single-sideband; hence, if the lowpass channel has bandwidth from $-W$ to W , the passband channel will have bandwidth equal to W ; on the other hand, a PSK or QAM system will have passband bandwidth equal to $2W$). The number of bits per symbol is

$$k = \frac{14400}{4800} = 3$$

Hence, the number of transmitted symbols is $2^3 = 8$. If a duobinary pulse is used for transmission, then the number of possible transmitted symbols is $2M - 1 = 15$. These symbols have the form

$$b_n = 0, \pm 2d, \pm 4d, \dots, \pm 12d$$

where $2d$ is the minimum distance between the points of the 8-PAM constellation. The probability mass function of the received symbols is

$$P(b = 2md) = \frac{8 - |m|}{64}, \quad m = 0, \pm 1, \dots, \pm 7$$

An upper bound of the probability of error is given by (see (9-3-18))

$$P_M < 2 \left(1 - \frac{1}{M^2} \right) Q \left[\sqrt{\left(\frac{\pi}{4} \right)^2 \frac{6}{M^2 - 1} \frac{k\mathcal{E}_{b,av}}{N_0}} \right]$$

With $P_M = 10^{-6}$ and $M = 8$ we obtain

$$\frac{k\mathcal{E}_{b,av}}{N_0} = 1.3193 \times 10^3 \implies \mathcal{E}_{b,av} = 0.088$$

Problem 9.21 :

(a) The spectrum of the baseband signal is (see (4-4-12))

$$\Phi_V(f) = \frac{1}{T} \Phi_{ii}(f) |X_{rc}(f)|^2 = \frac{1}{T} |X_{rc}(f)|^2$$

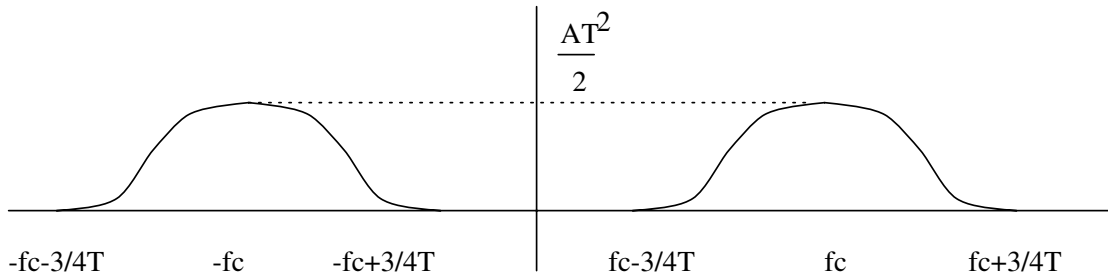
where $T = \frac{1}{2400}$ and

$$X_{rc}(f) = \begin{cases} T & 0 \leq |f| \leq \frac{1}{4T} \\ \frac{T}{2}(1 + \cos(2\pi T(|f| - \frac{1}{4T}))) & \frac{1}{4T} \leq |f| \leq \frac{3}{4T} \\ 0 & \text{otherwise} \end{cases}$$

If the carrier signal has the form $c(t) = A \cos(2\pi f_c t)$, then the spectrum of the DSB-SC modulated signal, $\Phi_U(f)$, is

$$\Phi_U(f) = \frac{A}{2} [\Phi_V(f - f_c) + \Phi_V(f + f_c)]$$

A sketch of $\Phi_U(f)$ is shown in the next figure.



(b) Assuming bandpass coherent demodulation using a matched filter, the received signal $r(t)$ is first passed through a linear filter with impulse response

$$g_R(t) = A x_{rc}(T - t) \cos(2\pi f_c(T - t))$$

The output of the matched filter is sampled at $t = T$ and the samples are passed to the detector. The detector is a simple threshold device that decides if a binary 1 or 0 was transmitted depending on the sign of the input samples. The following figure shows a block diagram of the optimum bandpass coherent demodulator.

