EE 163
Communication Theory I

Winter 2003

http://ee163.caltech.edu

Homework 6

1. Consider the following detection problem:

$$r(t) = s_m(t) + n(t) \qquad 0 \le t \le T$$

for $m = 1, 2, \ldots, 8$, where

$$s_m(t) = \sqrt{2E/T}\sin\left(t + m\frac{\pi}{4}\right)$$

with E being the symbol energy.

This signal set is known as 8-PSK. Assume the 8 signals are equally likely, and $n(t)$ is AWGN with PSD $N_0/2$ watts/Hz.

(a) Choose a suitable orthonormal set for the signal space. Plot the signal constellation. What is the dimensionality of this space?

(b) Compute the distances (as a function of E) between an arbitrary signal point in the constellation, and the seven other points.

(c) Indicate the optimum decision regions on your signal constellation plot.

(d) Use the results from parts (b) and (c) to compute a union bound as an upper bound on the probability of symbol error.

2. (Problem 5.38) The discrete sequence

$$r_k = \sqrt{E_b}c_k + n_k \qquad k = 1, 2, \ldots, n$$

represents the output sequence of samples from a demodulator, where $c_k = \pm 1$ are elements of one of two possible input code words; $\vec{C}_1 = [1\ 1\ 1 \cdots 1]$ and $\vec{C}_2 = [1\ 1\ 1 \cdots 1\ \ -1\ \ -1 \cdots -1]$. The code word $\vec{C}_2$ has w elements that are $+1$ and $n - w$ elements that are -1, where w is some positive integer. The noise sequence $\{n_k\}$ is white Gaussian with variance σ^2.

(a) What is the optimum maximum likelihood detector for the two possible transmitted signals, i.e., what is the optimum decision rule?

(b) Determine the probability of error as a function of all or some of the parameters σ^2, E_b, w, and n.

(c) What is the value of w that minimizes the error probability?

3. (Problem 5.42) In on-off keying of a carrier modulated signal, the two possible equiprobable signals are

$$\begin{aligned} s_0(t) &= 0 \qquad 0 \le t \le T_b \\ s_1(t) &= \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \qquad 0 \le t \le T_b \end{aligned}$$

The corresponding received signals are

$$\begin{aligned} r(t) &= n(t), \qquad 0 \le t \le T_b \\ r(t) &= \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \varphi) + n(t), \qquad 0 \le t \le T_b \end{aligned}$$

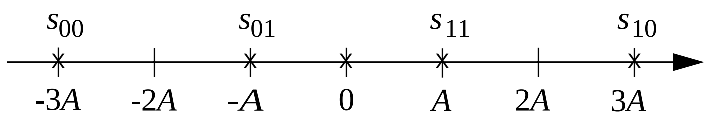
where φ is the carrier phase and $n(t)$ is AWGN.

(a) Sketch a block diagram of the receiver (demodulator and detector) that employs noncoherent (envelope) detection. Assume the demodulator requires at least one oscillator (sinusoidal generator).

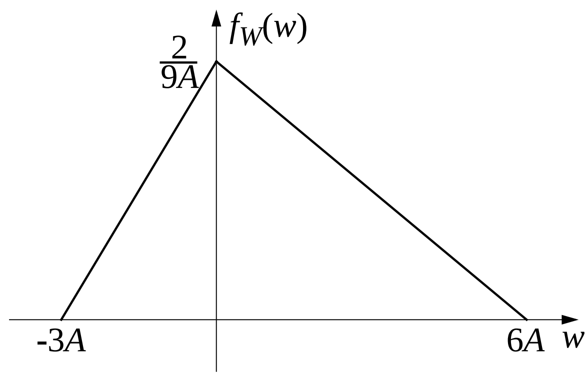
(b) Determine the probability density functions for the two possible decision variables at the detector corresponding to the two possible received signals.

(c) Write out the general integral expression for the probability of error, which may be a function of one or more Bessel functions.

4. Suppose the following signal constellation is used for transmission:



Assume that the sampled noise at the output of the matched filter has the following probability density function:



$$f_W(w) = \begin{cases} 0, & w < -3A \\ \dfrac{2w+6A}{27A^2}, & -3A \le w < 0 \\ \dfrac{-w+6A}{27A^2}, & 0 \le w < 6A \\ 0, & w \ge 6A \end{cases}$$

(a) Find the optimum decision rule for this scheme. Carefully draw the boundaries of the decision regions in a signal space diagram. Assume equiprobable signals.

(b) Find P_b, the average probability of a bit error.