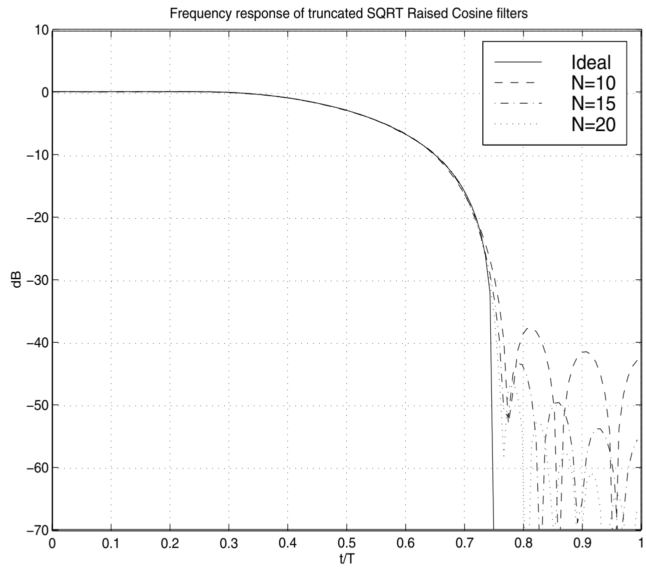
Problem 9.6 :

(a)(b) In order to calculate the frequency response based on the impulse response, we need the values of the impulse response at $t = 0, \pm T/2$, which are not given directly by the expression of Problem 9.5. Using L'Hospital's rule it is straightforward to show that:

$$x(0) = \frac{1}{2} + \frac{2}{\pi}, \quad x(\pm T/2) = \frac{\sqrt{2}}{2} \frac{(2+\pi)}{2\pi}$$

Then, the frequency response of the filters with $N = 10, 15, 20$ compared to the frequency response of the ideal square-root raised cosine filter are depicted in the following figure.



As we see, there is no significant difference in the passband area of the filters, but the realizable, truncated filters do have spectral sidelobes outside their $(1 + \beta)/T$ nominal bandwidth. Still, depending on how much residual ISI an application can tolerate, even the $N = 10$ filter appears an acceptable approximation of the ideal (non-realizable) square-root raised cosine filter.

Problem 9.10 :

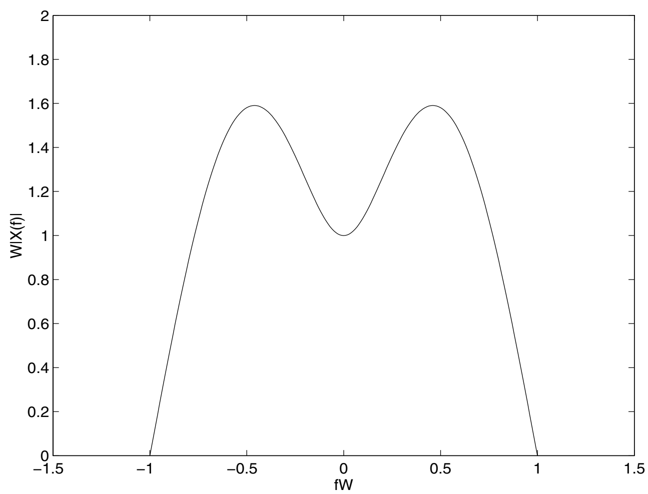
(a)
(i) $x_0 = 2,\ x_1 = 1,\ x_2 = -1,$ otherwise $x_n = 0$. Then :

$$x(t) = 2\frac{\sin(2\pi W t)}{2\pi W t} + \frac{\sin(2\pi W(t-1/2W))}{2\pi W(t-1/2W)} - \frac{\sin(2\pi W(t-1/W))}{2\pi W(t-1/W)}$$

and :

$$X(f) = \tfrac{1}{2W}\left[2 + e^{-j\pi f/W} - e^{-j2\pi f/W}\right], \quad |f| \le W \Rightarrow$$
$$|X(f)| = \tfrac{1}{2W}\left[6 + 2\cos\tfrac{\pi f}{W} - 4\cos\tfrac{2\pi f}{W}\right]^{1/2}, \quad |f| \le W$$

The plot of $|X(f)|$ is given in the following figure :



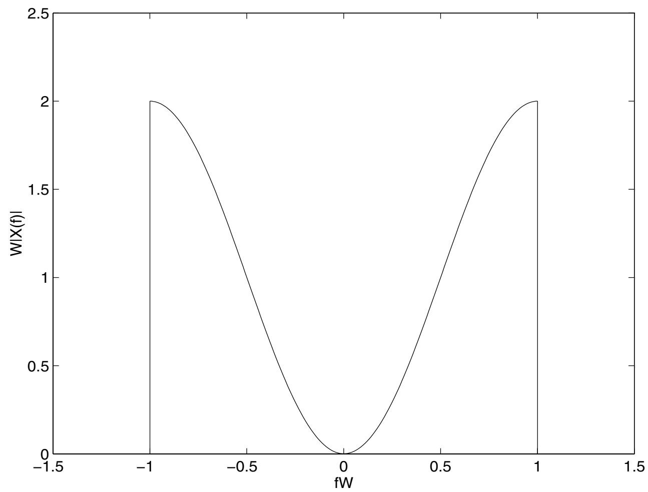
(ii) $x_{-1} = -1,\ x_0 = 2, x_1 = -1,$ otherwise $x_n = 0$. Then :

$$x(t) = 2\frac{\sin(2\pi W t)}{2\pi W t} - \frac{\sin(2\pi W(t+1/2W))}{2\pi W(t+1/2W)} - \frac{\sin(2\pi W(t-1/2W))}{2\pi W(t-1/2W)}$$

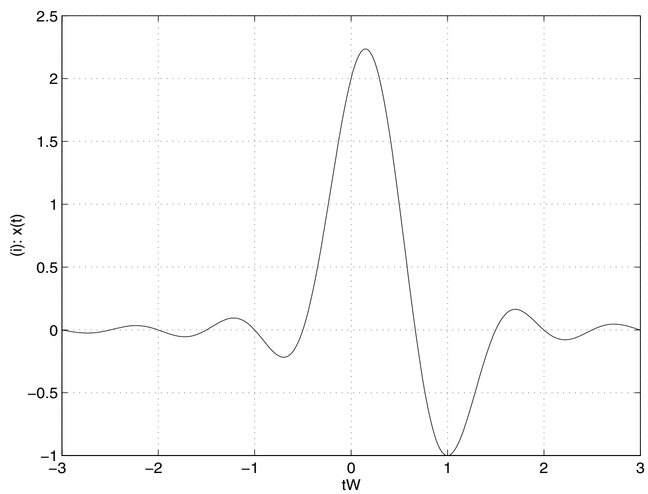
and :

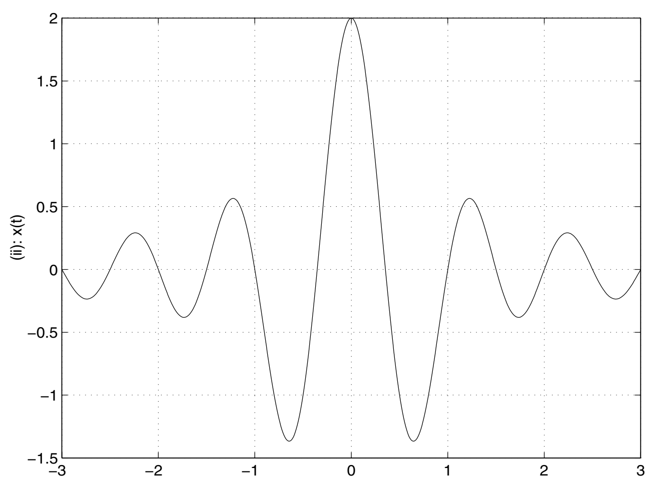
$$X(f) = \frac{1}{2W}\left[2 - e^{-j\pi f/W} - e^{+j\pi f/W}\right] = \frac{1}{2W}\left[2 - 2\cos\frac{\pi f}{W}\right] = \frac{1}{W}\left[1 - \cos\frac{\pi f}{W}\right],\ |f| \le W$$

The plot of $|X(f)|$ is given in the following figure :



(b) Based on the results obtained in part (a) :





(c) The possible received levels at the receiver are given by :
(i)

$$B_n = 2I_n + I_{n-1} - I_{n-2}$$

where $I_m = \pm 1$. Hence :

$$\begin{gathered} P(B_n = 0) = 1/4 \\ P(B_n = -2) = 1/4 \\ P(B_n = 2) = 1/4 \\ P(B_n = -4) = 1/8 \\ P(B_n = 4) = 1/8 \end{gathered}$$

(ii)

$$B_n = 2I_n - I_{n-1} - I_{n+1}$$

where $I_m = \pm 1$. Hence :

$$\begin{gathered} P(B_n = 0) = 1/4 \\ P(B_n = -2) = 1/4 \\ P(B_n = 2) = 1/4 \\ P(B_n = -4) = 1/8 \\ P(B_n = 4) = 1/8 \end{gathered}$$

Problem 9.12 :

The channel (bandpass) bandwidth is $W = 4000$ Hz. Hence, the lowpass equivalent bandwidth will extend from -2 to 2 KHz.
(a) Binary PAM with a pulse shape that has $\beta = \frac{1}{2}$. Hence :

$$\frac{1}{2T}(1+\beta) = 2000$$

so $\frac{1}{T} = 2667$, and since $k = 1$ bit/symbols is transmitted, the bit rate is 2667 bps.
(b) Four-phase PSK with a pulse shape that has $\beta = \frac{1}{2}$. From (a) the symbol rate is $\frac{1}{T} = 2667$ and the bit rate is 5334 bps.
(c) $M = 8$ QAM with a pulse shape that has $\beta = \frac{1}{2}$. From (a), the symbol rate is $\frac{1}{T} = 2667$ and hence the bit rate $\frac{3}{T} = 8001$ bps.

(d) Binary FSK with noncoherent detection. Assuming that the frequency separation between the two frequencies is $\Delta f = \frac{1}{T}$, where $\frac{1}{T}$ is the bit rate, the two frequencies are $f_c + \frac{1}{2T}$ and $f_c - \frac{1}{2T}$. Since $W = 4000$ Hz, we may select $\frac{1}{2T} = 1000$, or, equivalently, $\frac{1}{T} = 2000$. Hence, the bit rate is 2000 bps, and the two FSK signals are orthogonal.
(e) Four FSK with noncoherent detection. In this case we need four frequencies with separation of $\frac{1}{T}$ between adjacent frequencies. We select $f_1 = f_c - \frac{1.5}{T}$, $f_2 = f_c - \frac{1}{2T}$, $f_3 = f_c + \frac{1}{2T}$, and $f_4 = f_c + \frac{1.5}{T}$, where $\frac{1}{2T} = 500$ Hz. Hence, the symbol rate is $\frac{1}{T} = 1000$ symbols per second and since each symbol carries two bits of information, the bit rate is 2000 bps.
(f) $M = 8$ FSK with noncoherent detection. In this case we require eight frequencies with frequency separation of $\frac{1}{T} = 500$ Hz for orthogonality. Since each symbol carries 3 bits of information, the bit rate is 1500 bps.

Problem 9.14 :

The bandwidth of the bandpass channel is :

$$W = 3300 - 300 = 3000 \text{ Hz}$$

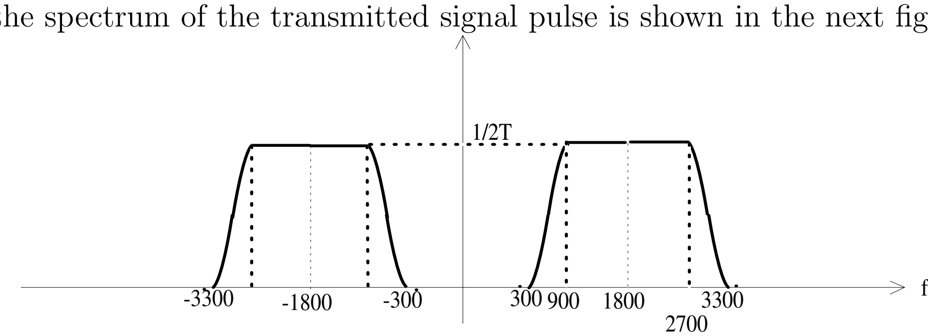
In order to transmit 9600 bps with a symbor rate $R = \frac{1}{T} = 2400$ symbols per second, the number of information bits per symbol should be :

$$k = \frac{9600}{2400} = 4$$

Hence, a $2^4 = 16$ QAM signal constellation is needed. The carrier frequency f_c is set to 1800 Hz, which is the mid-frequency of the frequency band that the bandpass channel occupies. If a pulse with raised cosine spectrum and roll-off factor β is used for spectral shaping, then for the bandpass signal with bandwidth W :

$$\frac{1}{2T}(1+\beta) = \frac{W}{2} = 1500 \Rightarrow \beta = 0.25$$

A sketch of the spectrum of the transmitted signal pulse is shown in the next figure.



Problem 9.20 :

Since the one-sided bandwidth of the ideal lowpass channel is $W = 2400$ Hz, the rate of transmission is :

$$R = 2 \times 2400 = 4800 \text{ symbols/sec}$$

(remember that PAM can be transmitted single-sideband; hence, if the lowpass channel has bandwidth from -W to W, the passband channel will have bandwidth equal to W; on the other hand, a PSK or QAM system will have passband bandwidth equal to $2W$). The number of bits per symbol is

$$k = \frac{14400}{4800} = 3$$

Hence, the number of transmitted symbols is $2^3 = 8$. If a duobinary pulse is used for transmission, then the number of possible transmitted symbols is $2M - 1 = 15$. These symbols have the form

$$b_n = 0, \pm 2d, \pm 4d, \ldots, \pm 12d$$

where $2d$ is the minimum distance between the points of the 8-PAM constellation. The probability mass function of the received symbols is

$$P(b = 2md) = \frac{8 - |m|}{64}, \qquad m = 0, \pm 1, \ldots, \pm 7$$

An upper bound of the probability of error is given by (see (9-3-18))

$$P_M < 2\left(1 - \frac{1}{M^2}\right) Q\left[\sqrt{\left(\frac{\pi}{4}\right)^2 \frac{6}{M^2 - 1} \frac{k\mathcal{E}_{b,av}}{N_0}}\right]$$

With $P_M = 10^{-6}$ and $M = 8$ we obtain

$$\frac{k\mathcal{E}_{b,av}}{N_0} = 1.3193 \times 10^3 \Longrightarrow \mathcal{E}_{b,av} = 0.088$$

Problem 9.21 :

(a) The spectrum of the baseband signal is (see (4-4-12))

$$\Phi_V(f) = \frac{1}{T}\Phi_i i(f)|X_{rc}(f)|^2 = \frac{1}{T}|X_{rc}(f)|^2$$

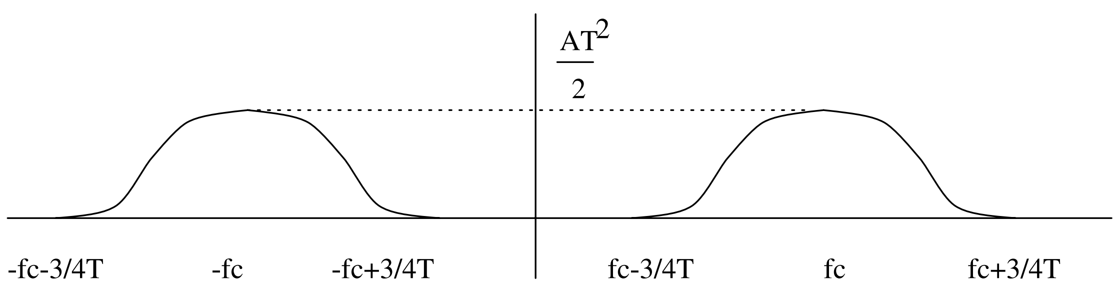
where $T = \frac{1}{2400}$ and

$$X_{rc}(f) = \begin{cases} T & 0 \leq |f| \leq \frac{1}{4T} \\ \frac{T}{2}(1 + \cos(2\pi T(|f| - \frac{1}{4T})) & \frac{1}{4T} \leq |f| \leq \frac{3}{4T} \\ 0 & \text{otherwise} \end{cases}$$

If the carrier signal has the form $c(t) = A\cos(2\pi f_c t)$, then the spectrum of the DSB-SC modulated signal, $\Phi_U(f)$, is

$$\Phi_U(f) = \frac{A}{2}[\Phi_V(f - f_c) + \Phi_V(f + f_c)]$$

A sketch of $\Phi_U(f)$ is shown in the next figure.



(b) Assuming bandpass coherent demodulation using a matched filter, the received signal $r(t)$ is first passed through a linear filter with impulse response

$$g_R(t) = Ax_{rc}(T - t)\cos(2\pi f_c(T - t))$$

The output of the matched filter is sampled at $t = T$ and the samples are passed to the detector. The detector is a simple threshold device that decides if a binary 1 or 0 was transmitted depending on the sign of the input samples. The following figure shows a block diagram of the optimum bandpass coherent demodulator.