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Problem 10.10 :

(a) The equivalent discrete-time impulse response of the channel is :

$$h(t) = \sum_{n=-1}^{1} h_n \delta(t - nT) = 0.3\delta(t + T) + 0.9\delta(t) + 0.3\delta(t - T)$$

If by $\{c_n\}$ we denote the coefficients of the FIR equalizer, then the equalized signal is :

$$q_m = \sum_{n=-1}^{1} c_n h_{m-n}$$

which in matrix notation is written as :

$$\begin{pmatrix} 0.9 & 0.3 & 0. \\ 0.3 & 0.9 & 0.3 \\ 0. & 0.3 & 0.9 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

The coefficients of the zero-force equalizer can be found by solving the previous matrix equation. Thus,

$$\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} -0.4762 \\ 1.4286 \\ -0.4762 \end{pmatrix}$$

(b) The values of q_m for $m = \pm 2, \pm 3$ are given by

$$\begin{aligned} q_2 &= \sum_{n=-1}^{1} c_n h_{2-n} = c_1 h_1 = -0.1429 \\ q_{-2} &= \sum_{n=-1}^{1} c_n h_{-2-n} = c_{-1} h_{-1} = -0.1429 \\ q_3 &= \sum_{n=-1}^{1} c_n h_{3-n} = 0 \\ q_{-3} &= \sum_{n=-1}^{1} c_n h_{-3-n} = 0 \end{aligned}$$

Problem 10.11 :

(a) The output of the zero-force equalizer is :

$$q_m = \sum_{n=-1}^{1} c_n x_{m_n}$$

With $q_0 = 1$ and $q_m = 0$ for $m \neq 0$, we obtain the system :

$$\begin{pmatrix} 1.0 & 0.1 & -0.5 \\ -0.2 & 1.0 & 0.1 \\ 0.05 & -0.2 & 1.0 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Solving the previous system in terms of the equalizer's coefficients, we obtain :

$$\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0.000 \\ 0.980 \\ 0.196 \end{pmatrix}$$

(b) The output of the equalizer is :

$$q_m = \begin{cases} 0 & m \leq -4 \\ c_{-1}x_{-2} = 0 & m = -3 \\ c_{-1}x_{-1} + c_0 x_{-2} = -0.49 & m = -2 \\ 0 & m = -1 \\ 1 & m = 0 \\ 0 & m = 1 \\ c_0 x_2 + x_1 c_1 = 0.0098 & m = 2 \\ c_1 x_2 = 0.0098 & m = 3 \\ 0 & m \geq 4 \end{cases}$$

Hence, the residual ISI sequence is

$$\text{residual ISI} = \{\ldots, 0, -0.49, 0, 0, 0, 0.0098, 0.0098, 0, \ldots\}$$

and its span is 6 symbols.

Problem 10.13 :

The optimum tap coefficients of the zero-force equalizer can be found by solving the system:

$$\begin{pmatrix} 1.0 & 0.3 & 0.0 \\ 0.2 & 1.0 & 0.3 \\ 0.0 & 0.2 & 1.0 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Hence,

$$\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} -0.3409 \\ 1.1364 \\ -0.2273 \end{pmatrix}$$

The output of the equalizer is :

$$q_m = \begin{cases} 0 & m \leq -3 \\ c_{-1}x_{-1} = -0.1023 & m = -2 \\ 0 & m = -1 \\ 1 & m = 0 \\ 0 & m = 1 \\ c_1 x_1 = -0.0455 & m = 2 \\ 0 & m \geq 3 \end{cases}$$

Hence, the residual ISI sequence is :

$$\text{residual ISI} = \{\ldots, 0, -0.1023, 0, 0, 0, -0.0455, 0, \ldots\}$$

Problem 10.18 :

(a) $X(z) = F(z)F^*(z^{-1}) = \frac{1}{2}z + 1 + \frac{1}{2}z^{-1}$. Then, the covariance matrix $\mathbf{\Gamma}$ is :

$$\mathbf{\Gamma} = \begin{bmatrix} 1+N_0 & 1/2 & 0 \\ 1/2 & 1+N_0 & 1/2 \\ 0 & 1/2 & 1+N_0 \end{bmatrix} \text{ and } \xi = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

The optimum equalizer coefficients are given by :

$$\begin{aligned} \mathbf{C}_{opt} &= \mathbf{\Gamma}^{-1}\xi \\ &= \frac{1}{\det(\mathbf{\Gamma})} \begin{bmatrix} (1+N_0)^2 - 1/4 & -\frac{1}{2}(1+N_0) & 1/4 \\ -\frac{1}{2}(1+N_0) & (1+N_0)^2 & -\frac{1}{2}(1+N_0) \\ 1/4 & -\frac{1}{2}(1+N_0) & (1+N_0)^2 - 1/4 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} \\ &= \frac{1}{\sqrt{2}\det(\mathbf{\Gamma})} \begin{bmatrix} N_0^2 + \frac{3}{2}N_0 + \frac{1}{4} \\ N_0^2 + \frac{3}{2}N_0 + \frac{1}{2} \\ -\frac{N_0}{2} - \frac{1}{4} \end{bmatrix} \end{aligned}$$

where $\det(\mathbf{\Gamma}) = (1+N_0)\left[(1+N_0)^2 - \frac{1}{2}\right]$

(b)

$$\det(\mathbf{\Gamma} - \lambda\mathbf{I}) = (1+N_0-\lambda)\left[(1+N_0-\lambda)^2 - \tfrac{1}{2}\right] \Rightarrow$$
$$\lambda_1 = 1+N_0,\ \lambda_2 = \tfrac{1}{\sqrt{2}} + 1 + N_0,\ \lambda_3 = 1 - \tfrac{1}{\sqrt{2}} + N_0$$

and the corresponding eigenvectors are :

$$\mathbf{v}_1 = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix},\ \mathbf{v}_2 = \begin{bmatrix} 1/2 \\ 1/\sqrt{2} \\ 1/2 \end{bmatrix},\ \mathbf{v}_3 = \begin{bmatrix} 1/2 \\ -1/\sqrt{2} \\ 1/2 \end{bmatrix}$$

(c)

$$J_{\min}(K)|_{K=1} = J_{\min}(1) = 1 - \xi'\Gamma^{-1}\xi = \frac{2N_0^3 + 4N_0^2 + 2N_0 + 3/4}{2N_0^3 + 4N_0^2 + 5N_0 + 1}$$

(d)

$$\gamma = \frac{1 - J_{\min}(1)}{J_{\min}(1)} = \frac{2N_0^2 + 3N_0 + 3/4}{2N_0^3 + 4N_0^2 + 1/4}$$

Note that as $N_0 \to 0$, $\gamma \to 3$. For $N_0 = 0.1$, $\gamma = 2.18$ for the 3-tap equalizer and $\gamma = \sqrt{1 + \frac{2}{N_0}} - 1 = 3.58$, for the infinite-tap equalizer (as in example 10-2-1). Also, note that $\gamma = \frac{1}{N_0} = 10$ for the case of no intersymbol interference.